All Subjects — All Topics — PYQ: CSIR Net July 2026

75 approved questions

Questions

Q1. Three alarm bells ring at intervals $30\sec$, $40\sec$ and 45 sec respectively. They started ringing simultaneously at 4:00 am in the morning.
At what time will they all ring simultaneously:

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(A) 4:06 am
(B) 4:06 am
(C) 4:40 am
(D) 5:00 am

Q2. In a certain code if ' 1000 ' is written as ' 1728 ' and ' 125 ' is written as ' 343 '. And ' 343 ' is written as ' 729 ', in the same code ' 512 ' will be written as:

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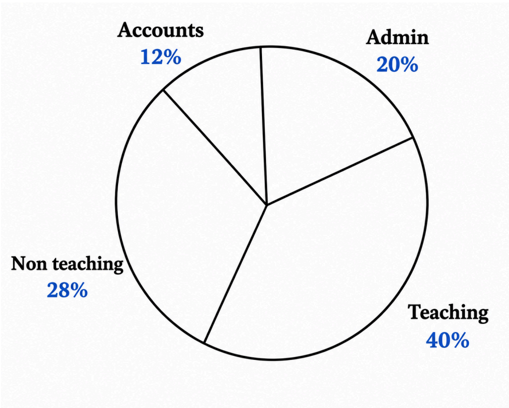
(A) 1000
(B) 1331
(C) 990
(D) 1010

Q3. The pie chart shows the percentage of employees working in different departments of a University. The table shows the ratio of males to females in each of these departments.

If the total number of employees $= 2000$, what is the ratio between female employees in Admin and non teaching departments?

Category	Male : Female
Accounts	7 : 5
Admin	2 : 3
Teaching	5 : 3
Non teaching	3 : 4



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(A) 3 : 4

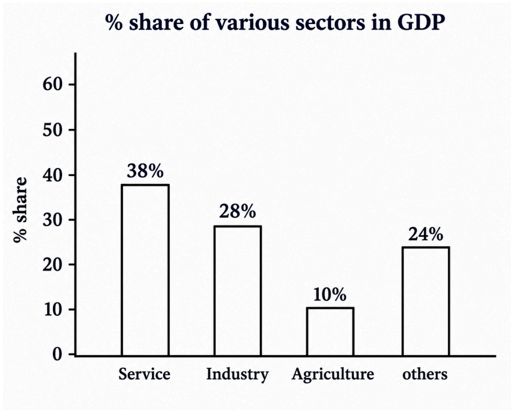
(B) 5 : 3

(C) 2:3

(D) 4 : 3

Q4. The following chart shows the sector wise percentage break up of Rs. 650 crore, which is the GDP of a small country with a population of 1 billion people. 80% of the population constitutes its working population engaged in various sector and contributing to GDP.

Which of the following has lowest contribution to GDP?



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(A) Service

(B) Industry

(C) Agriculture

(D) Non working population

Q5. Candidate in a competitive examination consisted of 60% men and 40% woman. 70% of men and 75% of women cleared the qualifying test and entered the final test where 80% men and 70% women were successful. Which of the following statement is correct?

1. Overall success rate is below 50%
2. Success rate is higher for women
3. More men cleared the examination than women
4. Both 1 and 2

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(A) Overall success rate is below 50%

(B) Success rate is higher for women

(C) More men cleared the examination than women

(D) Both 1 and 2

Q6. There are four positive integers say $\mathrm{A}, \mathrm{B}, \mathrm{C}$ and D such that $\mathrm{A} \leq \mathrm{B} \leq \mathrm{C} \leq \mathrm{D}$. Consider the following statements :

A. The unique mode of the data is 6.

B. The median score of the data is 7.

What is the minimum possible value of the largest number D ?

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(A) 6
(B) 7
(C) 8
(D) 9

Q7. A certain sum of money becomes double of itself in 5 years at simple rate of interest. In how many years will it become 16 times of itself at the same rate of simple interest?

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(A) 80
(B) 70
(C) 75
(D) 90

Q8. XUR, TQN, PMJ, _____ ?, HEB.

Which of the following options will complete the above series by replacing the question mark?

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(A) KHE
(B) LIF
(C) LJH
(D) MJG

Q9. During the interaction of population of two different species identify the type where only one species is benefited and is detrimental to the other species.

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(A) Commensalism and Amensalism
(B) Commensalism and Competition
(C) Parasitism and Predation
(D) Parasitism and Amenalism

Q10. Consider the relationship statement between two natural numbers :

A. the difference between them is 6.
B. the difference between their squares is 60.

Find the sum of the cubes of these two numbers.

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(A) 620
(B) 520
(C) 720
(D) 820

Q11. There are four towns $\mathrm{P}, \mathrm{Q}, \mathrm{R}$ and $\mathrm{T} . \mathrm{Q}$ is to the South-West of P, R is to the East of Q and South-East of P and T is to the North of R in line with Q, P. In which direction of P is T located?

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(A) South-East
(B) North
(C) North-East
(D) East

Q12. In which of the following, entropy will decrease?

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(A) Water is boiled
(B) Sugar is dissolved in water
(C) Ice is heated to melt
(D) Egg is boiled in water

Q13. A process is used to separate substances in chemical mixtures. What is this process known as?

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(A) Distillation
(B) Filtration
(C) Chromatography
(D) Crystallisation

Q14. Suppose there are ' 9 ' identical gold-coins out of which 8 coins have same weight. One of the coin is counterfeit and weighs slightly less than other coins. You have a classical balance scale for weighing.
What is the minimum number of weighings required to definitively isolate and identify the counterfeit coin?

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(A) 3
(B) 4
(C) 2
(D) 8

Q15. The largest possible triangle is inscribed in a semi-circle of diameter 14 cm. What would be the area inside the semi-circle which is not occupied by the triangle?

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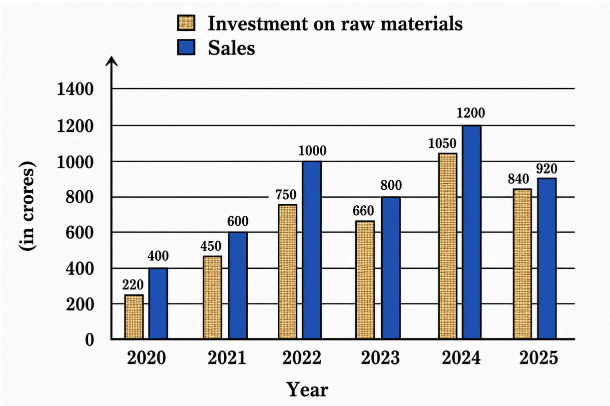
(A) 35 sq. cm
(B) 30 sq. cm
(C) 28 sq. cm
(D) 48 sq. cm

Q16. The graph represents investment on raw material and sales of an organization from 2020 to 2025 (calendar year) Study the bar graph and answer the question.

In which year was there a maximum percentage increase in the sales of products as compared the previous year?



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(A) 2021
(B) 2023
(C) 2022
(D) 2024

Q17. Read the following information to answer the question

$\mathrm{P}+\mathrm{Q}$ means P is the brother of Q ;

$\mathrm{P}\times\mathrm{Q}$ mean P is the father of Q ;

$\mathrm{P}-\mathrm{Q}$ means P is the sister of Q .

Which of the following relation shows that I is the niece of K?

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(A) $\mathrm{K}+\mathrm{Y}+\mathrm{Z}-\mathrm{I}$
(B) $\mathrm{K}+\mathrm{Y}\times\mathrm{I}-\mathrm{Z}$
(C) $\mathrm{Z}-\mathrm{I}\times\mathrm{Y}+\mathrm{K}$
(D) $\mathrm{K}\times\mathrm{Y}+\mathrm{I}-\mathrm{Z}$

Q18. There are two numbers such that the difference between them is 10 and their ratio is $5:3$. Find the product of these two numbers

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(A) 150
(B) 375
(C) 275
(D) 80

Q19. The present population of a city is 6000 . If the number of males increases by 14% and the number of females increases by 22%, then the population will become 6900 . What is the present population of females in the city?

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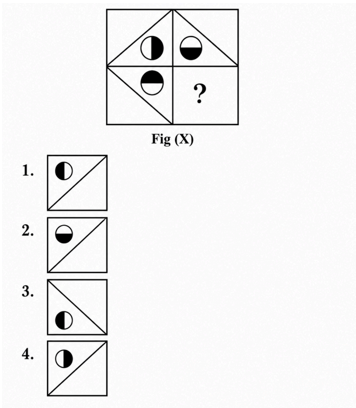
(A) 900
(B) 700
(C) 750
(D) 850

Q20. Select a figure from amongst the four alternatives which when placed in the blank space of fig (X) would complete the pattern?



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(A) 1
(B) 2
(C) 3
(D) 4

Q21. An analog-to-digital converter monitors the dc voltage output from a battery. The output of the converter is a three-digit binary number ABC, corresponding to battery voltage in steps of 0.5 V, with A as the most significant bit (MSB). The output of the converter is fed into a logic circuit which produces a HIGH output when the battery voltage exceeds 2 V. The expression for this logic circuit is:

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(A) AB+AC
(B) $\mathrm{AB}+\mathrm{BC}+\mathrm{AC}$
(C) ABC
(D) AC

Q22. Consider the Poisson's equation $\nabla^2 V = -\frac{q}{\epsilon_0}\delta^3(\vec{r}-\vec{a})$ in half-space $z>0$ with boundary condition $V=0$ in the $z=0$ plane. Here, $\vec{a}=a_0\hat{k}$ with $a_0>0$. If $\frac{q}{4\pi\epsilon_0 a_0}=24\text{Volt}$, then which of the following is correct?

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(A) At $\vec{r}=(0,0,2a_0)\,, V=16\text{Volt}$

(B) At $\vec{r}=(0,0,3a_0)\,, V=8$ Volt

(C) At $\vec{r}=(0,0,4a_0)\,, V=4$ Volt

(D) At $\vec{r}=(0,0,a_0/2)\,, V=32$ Volt

Q23. You are given a time-independent observable O for a system with time independent Hamiltonian H and a general time dependent state $|\phi(t)\rangle$. If the expectation value $\langle O(t)\rangle_\phi = \langle\phi(t)|O|\phi(t)\rangle$, then consider the following statements (A) to (D) and choose the correct option.

(A) If O commutes with H then $\langle O(t)\rangle_\phi$ is independent of time.

(B) If $|\phi(t)\rangle$ is an eigen state of H then $\langle O(t)\rangle_\phi$ is independent of time.

(C) If $|\phi(t)\rangle$ is an eigen state of O then $\langle O(t)\rangle_\phi$ is independent of time.

(D) If $|\phi(t)\rangle$ is an eigen state of $[O,H]$ then $\langle O(t)\rangle_\phi$ is independent of time.

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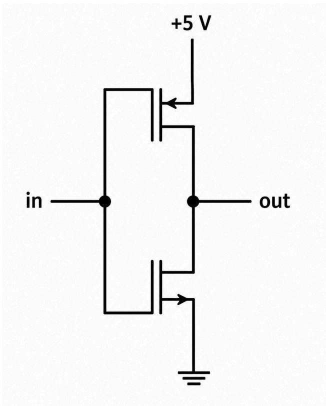
(A) Only (A) and (B) are correct

(B) Only (A) is correct

(C) Only (B), (C) and (D) are correct

(D) Only (A), (B) and (C) are correct

Q24. The circuit shown below could be used as a:



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(A) TTL NOT Gate

(B) TTL Buffer

(C) CMOS NOT Gate

(D) CMOS Buffer

Q25. A particle is in the ground state of a one-dimensional infinite well potential with walls at $x=0$ and at $x=a$. The wall at $x=a$ is moved to $x=2a$ in two separate, sudden and adiabatic, protocols so that $\langle E\rangle_s$ and $\langle E\rangle_a$ are the respective expectation values of the energy in the final states. Then the ratio $\frac{\langle E\rangle_s}{\langle E\rangle_a}$ is :

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(A) 4:1
(B) 2:1
(C) 1:1
(D) 1:4

Q26. Consider two operators $\widehat{R}$ and $\widehat{S}$ where $\widehat{R}$ commutes with $[\widehat{R}, \widehat{S}]$. If α is a constant, then $\left[e^{\alpha \hat{R}}, \hat{S}\right]$ is:

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(A) $\left[\hat{R}^{\alpha}, \hat{S}\right]$
(B) $\alpha e^{\alpha \hat{R}}[\hat{R}, \hat{S}]$
(C) 0
(D) $\alpha e^{\hat{R}}\left[e^{\alpha} \hat{R}, \hat{S}\right]$

Q27. A linearly polarized electromagnetic wave is incident normally on a material of refractive index n_R for right circular polarization and n_L for left circular polarization, where $n_R \neq n_L$. The reflected wave's polarization is :

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(A) Linear
(B) Elliptical
(C) Circular
(D) Random

Q28. In a 4 -dimensional vector space V_4, three linearly independent vectors are given by $\boldsymbol{e_1}=(1,1,0,0), \boldsymbol{e}_2=(0,1,1,0)$ and $\boldsymbol{e}_3=(0,0,1,1)$. Which of the following vector $\boldsymbol{e_4}$ will make $\{\boldsymbol{e_1}, \boldsymbol{e_2}, \boldsymbol{e_3}, \boldsymbol{e_4}\}$ a basis of V_4 ?

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(A) (1,1,1,1)
(B) (1,1,1,0)
(C) (1,0,-1,0)
(D) (0,1,1,0)

Q29. A micro-canonical ensemble consists of six non-interacting and distinguishable spin 1 particles in a uniform magnetic field. Each particle can thus be in energy states $-E_0, 0$ and E_0. The number of microstates in the state with total energy zero is:

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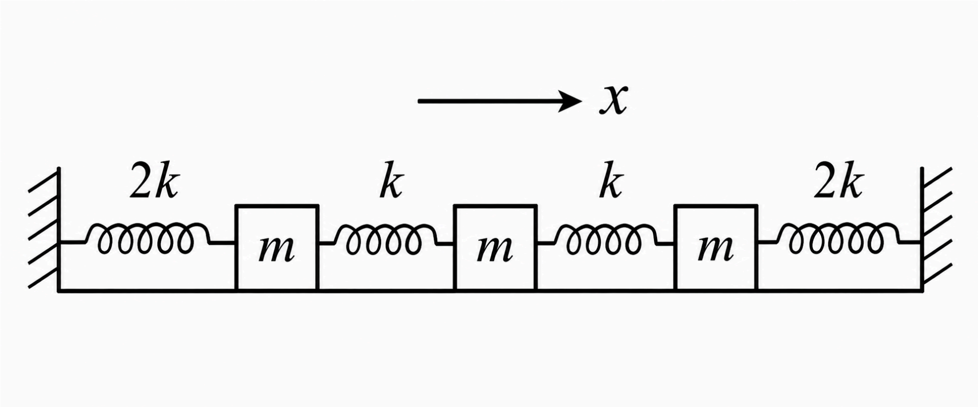
(A) 51
(B) 191
(C) 141
(D) 55

Q30. Three equal masses m are free to move along x-axis on a frictionless surface. The masses are coupled together, and to a rigid wall, by springs as shown in the figure. If $\omega_0 = \sqrt{\frac{k}{m}}$, and the angular frequencies of the three normal modes of this system are ω_1, ω_2 and ω_3 then the value of $\omega_1^2 + \omega_2^2 + \omega_3^2$ is:



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(A) $9\omega_0^2$

(B) $6\omega_0^2$

(C) $10\omega_0^2$

(D) $8\omega_0^2$

Q31. Consider the spin matrix S_r projected along the radial direction in spherical polar coordinates (r, θ, φ). The probability that a measurement of S_r on the state $|\beta\rangle = \begin{pmatrix} \sin\frac{\theta}{2} \\ e^{i\varphi}\cos\frac{\theta}{2} \end{pmatrix}$ will yield the value $\frac{\hbar}{2}$ is:

QUANTUM MECHANICS • SPIN AND TOTAL ANGULAR MOMENTUM • MEDIUM • PYQ · CSIR NET JULY 2026

(A) $\sin^2\theta$

(B) $\cos^2\theta$

(C) $\frac{1}{2}$

(D) $\cos^2\frac{\theta}{2}$

Q32. Water, having density $1\mathrm{gm/cm^3}$, freezes into ice at 273 K temperature and 1 atm pressure. The latent heat of melting of ice is $334\ \mathrm{J/gm}$ and the density of ice is $0.917\mathrm{gm/cm^3}$. Then for the melting temperature (T_m) which of the following options is correct?

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(A) At 140 atm pressure, $T_m \simeq 274\ \mathrm{K}$.

(B) At 420 atm pressure, $T_m \simeq 274\ \mathrm{K}$.

(C) At 140 atm pressure, $T_m \simeq 272\ \mathrm{K}$.

(D) At 420 atm pressure, $T_m \simeq 272\ \mathrm{K}$.

Q33. A particle moves in an orbit $r = k\exp(\alpha\theta)$ under a central force $F(r)$. Here, k and α are constants and $k > 0$. Then $F(r)$ is proportional to:

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(A) $\frac{1}{r}$
(B) $\frac{1}{r^2}$
(C) $\frac{1}{r^3}$
(D) $\frac{1}{r^4}$

Q34. The Lagrangian of a system is given by $L = x^2 + x\dot{x} + \frac{\dot{x}^2}{2}$. If p is the generalized momentum, then the corresponding Hamiltonian is:

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(A) $\frac{\left(p^2 - x^2 - 2xp\right)}{2}$
(B) $\frac{\left(p^2 - x^2 + 2xp\right)}{2}$
(C) $\frac{\left(p^2 + x^2 + 2xp\right)}{2}$
(D) $\frac{\left(p^2 + x^2 - 2xp\right)}{2}$

Q35. A particle with charge q, which is confined to move in the xy-plane, is subjected to a magnetic field with magnitude B pointing in the $+z$ direction. The particle moves in a circle (C) of radius R. If $\vec{p}$ is the canonical momentum then the following integral

$$\oint_C \vec{p} \cdot d\vec{l}$$

has the magnitude:
($d\vec{l}$ is in the direction of motion)

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(A) $qB\pi R^2$
(B) $2qB\pi R^2$
(C) $3qB\pi R^2$
(D) $4qB\pi R^2$

Q36. A bead of mass M slides along a smooth frictionless and massless wire which is bent in the shape of a parabola $z = cr^2$ in cylindrical coordinates (r, ϕ, z). Here c is a positive constant. The wire is made to rotate about z-axis at constant angular velocity. Which of the following statements about this system of the wire and the bead is correct?

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(A) Energy is conserved but angular momentum is not conserved
(B) Energy is not conserved but angular momentum is conserved
(C) Both energy and angular momentum are conserved
(D) Both energy and angular momentum are not conserved

Q37. The value of the integral:

$$\oint \frac{d\theta}{5 - 4\cos\theta}$$

over the circle $|z| = 1$ in the anti-clockwise direction can be calculated by substituting $z = e^{i\theta}$. The value of the integral is:

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(A) $\frac{2\pi}{3}$
(B) $\frac{\pi}{3}$
(C) π
(D) $\frac{4\pi}{3}$

Q38. The eigen energies and the normalized eigenfunctions in the n^{th} state of a particle, confined in a one-dimensional infinite potential well, are denoted by E_n and ψ_n, respectively. If $E_1 = 1.2\text{eV}$ then the expectation value of energy in the state $\psi = 3\psi_1 + 2\psi_2 + \sqrt{3}\psi_4$ is close to:

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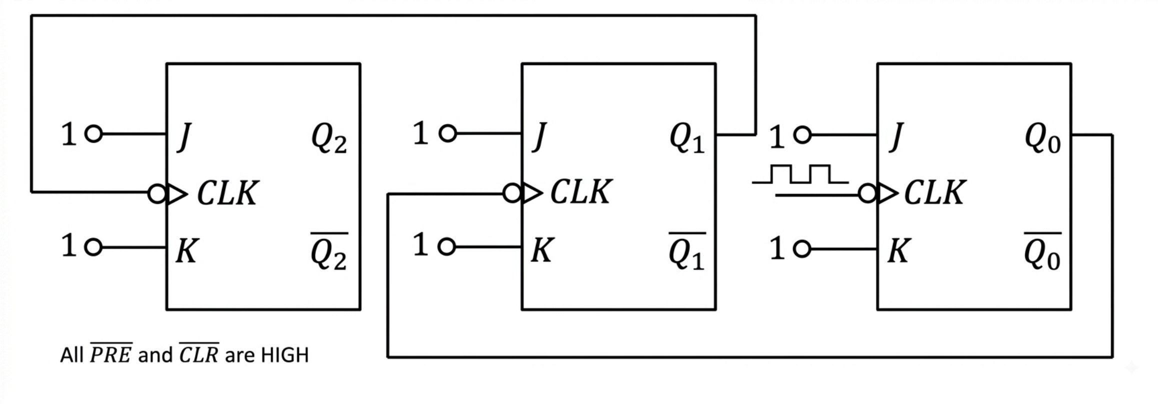
(A) 5.5 eV
(B) 5.0 eV
(C) 4.5 eV
(D) 6.0 eV

Q39. Three J-K flip-flops are connected together to make a counter as shown in the figure. The J and K inputs are tied to logical 1 and a clock pulse is applied only to CLK input of the flip flop Q_0. If the counter is initially in the state $Q_2Q_1Q_0 = 100$, then the state of the counter after 9 clock cycles is:



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(A) $Q_2Q_1Q_0 = 001$
(B) $Q_2Q_1Q_0 = 110$
(C) $Q_2Q_1Q_0 = 101$
(D) $Q_2Q_1Q_0 = 111$

Q40. A three-state system is described by a certain Hamiltonian having eigen energies $E_0, 2E_0$ and $3E_0$ with the corresponding eigen states $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$, $\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$, respectively. If the state of the system at time $t=0$ is $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, then what is the probability of finding the system in the same state at time $t=\frac{\pi h}{2E_0}$?

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(A) 1

(B) $\frac{1}{3}$

(C) $\frac{5}{18}$

(D) $\frac{2}{9}$

Q41. Vanadium has electron configuration $[Ar]3d^3 4s^2$. In LS coupling, its ground state configuration is

ATOMIC AND MOLECULAR PHYSICS • EFFECTS AND COUPLING IN ATOM • MEDIUM • PYQ · CSIR NET JULY 2026

(A) ${}^4F_9$

(B) ${}^2P_{\frac{1}{2}}$

(C) ${}^4F_{\frac{3}{2}}$

(D) ${}^4S_{\frac{3}{2}}$

Q42. In spherical polar coordinates (r, θ, ϕ), $\frac{\partial \hat{\theta}}{\partial \phi}$ is :

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(A) $\cos\theta \hat{\phi}$

(B) 0

(C) $\sin\theta \hat{\phi}$

(D) $-\cos\theta \hat{r}$

Q43. Internal energy of one mole of a non-ideal gas is given by $U=\frac{3}{2}RT-\frac{a}{V}$, where V is volume of the gas at temperature T and a is a positive constant. An insulated container of volume V_2 contains this gas confined in a volume V_1 at temperature T_1. The remaining volume V_2-V_1 of this container, which is in vacuum, is separated by a wall. The wall is suddenly removed which makes the gas volume V_2 and temperature T_2. The temperature T_2 is:
(R is the molar gas constant)

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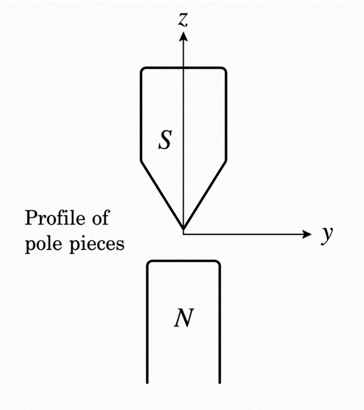
(A) $T_1+\frac{2a}{3R}\left(\frac{1}{V_2}+\frac{1}{V_1}\right)$

(B) $T_1-\frac{2a}{3R}\left(\frac{1}{V_2}+\frac{1}{V_1}\right)$

(C) $T_1+\frac{2a}{3R}\left(\frac{1}{V_2}-\frac{1}{V_1}\right)$

(D) $T_1-\frac{2a}{3R}\left(\frac{1}{V_2}-\frac{1}{V_1}\right)$

Q44. A beam of copper atoms $\left(\mathrm{Cu}:[\mathrm{Ar}]4\ \mathrm{s}^{1}3\ \mathrm{d}^{10}\right)$ is passed through a Stern-Gerlach setup (see figure). If the gradient of the z-component of the magnetic field in the z-direction is 10^{3} Tesla $/\mathrm{m}$, then the magnitude of the z-component of the force on an atom is:
(μ_B is the Bohr magneton)



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(A) $5\times10^{2}\mu_B$
(B) $10^{3}\mu_B$
(C) 0
(D) $2\pi\times10^{3}\mu_B$

Q45. Two particles occupy ten degenerate energy levels. There are three possible scenarios where the two particles are either identical bosons or identical fermions or distinguishable particles, with the associated number of accessible microstates denoted as $\Omega_B, \Omega_{\mathrm{F}}$ and Ω_C, respectively. Then, which of the following options is correct?

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(A) $\Omega_B=55, \Omega_F=45, \Omega_C=100$
(B) $\Omega_B=65, \Omega_F=55, \Omega_C=100$
(C) $\Omega_B=45, \Omega_F=25, \Omega_C=55$
(D) $\Omega_B=55, \Omega_F=45, \Omega_C=75$

Q46. The integral

$$\int_0^4 x^4 dx$$

when computed numerically using Simpson's $1/3^{\mathrm{rd}}$ method and trapezoidal method, each with unit step size, yields values I_s and I_t, respectively. Then which of the following options is correct?

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(A) $I_s=205.3, I_t=226.0$
(B) $I_s=205.3, I_t=205.3$
(C) $I_s=204.8, I_t=226.0$
(D) $I_s=226.0, \quad I_t=226.0$

Q47. Modulation techniques are used to shift a signal to higher frequencies to

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(A) remove the effects of white noise.

(B) reduce the effects of $1/f$-noise and drift.

(C) reduce shot noise and Johnson noise.

(D) smoothen the signal and amplify it.

Q48. A kaon (K^0) decays into two neutral pions (π^0). Given that the mass of the K^0 is $495 MeV/c^2$ and the mass of the π^0 is $135 MeV/c^2$, the final momentum of the π^0 (in MeV/c) in the center of mass frame is approximately:

NUCLEAR AND PARTICLE PHYSICS • PARTICLE PHYSICS • MEDIUM • PYQ · CSIR NET JULY 2026

(A) 207

(B) 359

(C) 157

(D) 103

Q49. In an experiment, a student generates 100 random numbers drawn from a uniform distribution in the interval $(0, 1)$ and calculates their mean. If the experiment is repeated 100 times, then the standard deviation of the means is:

EXPERIMENTAL TECHNIQUES • ERROR ANALYSIS • MEDIUM • PYQ · CSIR NET JULY 2026

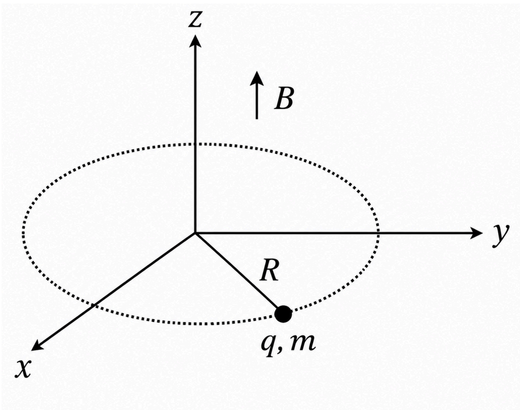
(A) $\frac{1}{20\sqrt{3}}$

(B) $\frac{1}{2\sqrt{3}}$

(C) $\frac{1}{\sqrt{3}}$

(D) $\frac{1}{200\sqrt{3}}$

Q50. A particle of mass m and charge q is fixed at one end of a rigid insulating rod of length R. The other end of the rod is pivoted such that it can rotate about the z-axis in the xy-plane as shown in the figure. A time dependent magnetic field $\vec{B} = \beta t \hat{z}$ is turned on at $t = 0$. Here β is a constant. Then the force in the rod at time t will be:



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(A) Compressive and of magnitude $q^2\beta^2t^2R/m$

(B) Zero

(C) Compressive and of magnitude $q^2\beta^2t^2R/(4m)$

(D) Tensile and of magnitude $q^2\beta^2t^2R/(4m)$

Q51. The Hamiltonian of a particle of mass m, moving in a circle of a fixed radius R, is given by $H_0 = \frac{L_z^2}{2mR^2}$, where $L_z = -i\hbar\frac{d}{d\phi}$ and ϕ is the azimuthal angle. This particle is perturbed by $H' = V\cos(2\phi)$ with $V \ll \frac{\hbar^2}{2mR^2}$. Within the first-order degenerate perturbation theory, the magnitude of the energy splitting of the first excited state is:

QUANTUM MECHANICS • PERTUBATION THEORY • MEDIUM • PYQ · CSIR NET JULY 2026

(A) $\frac{V}{2}$

(B) V

(C) $2V$

(D) $\frac{V}{4}$

Q52. For a one-dimensional system the Hamiltonian is given by $H = \frac{p^2}{2} - \frac{1}{2q^2}$. If a constant of motion of the system is $\left(apq + bp^2t + \frac{ct}{q^2}\right)$, where a, b and c are constants and t denotes the time, then which of the following options is correct:

CLASSICAL MECHANICS • POISSON BRACKET AND LAPLACE TRANSFORM • MEDIUM • PYQ · CSIR NET JULY 2026

(A) $a = b = c$

(B) $a = -b = c$

(C) $a = -b = -c$

(D) $a = b = -c$

Q53. There is a ferromagnetic interaction J between two Ising spins, $S_1, S_2 = \pm 1$, and they are under the influence of an external magnetic field h. The Hamiltonian of this system is,

$$H = -JS_1S_2 - hS_1 - hS_2.$$

The spins are in equilibrium at temperature T and $\beta = 1/k_BT$ with k_B as Boltzmann constant. The average value of either spin, for small h, is:

STATISTICAL MECHANICS • ISING MODEL • MEDIUM • PYQ · CSIR NET JULY 2026

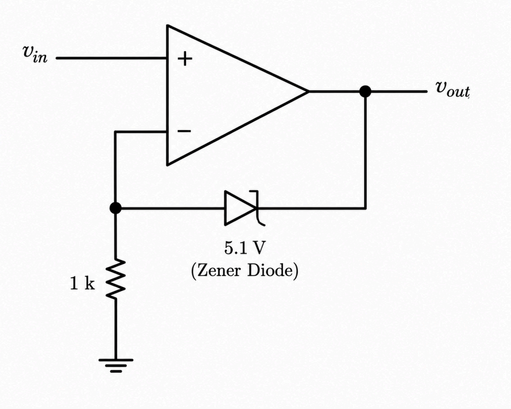
(A) $\frac{4h\exp(\beta J)}{\cosh(\beta J)}$

(B) $\frac{h\exp(2\beta J)}{\cos(\beta J)}$

(C) $\frac{8h\exp(4\beta J)}{\sinh(\beta J)}$

(D) $\frac{2h\exp(2\beta J)}{\sin(\beta J)}$

Q54. In the circuit shown below, if $V_{\text{in}} = 2.0\ \mathrm{V}$ then V_{out} is



ELECTRONICS • AMPLIFIERS AND OPAMP • MEDIUM • PYQ · CSIR NET JULY 2026

(A) 2.0 V
(B) 3.1 V
(C) 5.1 V
(D) 7.1 V

Q55. The Hamiltonian of a quantum mechanical system is given by $\widehat{H} = \alpha \widehat{L}^2 + \beta\hbar \widehat{L}_z$. Here, α and β are positive constants and $\widehat{L}$ represents the orbital angular momentum operator. If l and m are the azimuthal and magnetic quantum numbers then the lowest energy state for this system is:

QUANTUM MECHANICS • HYDROGEN ATOM (RADIAL AND ORBITAL ANGULAR MOMENTUM) • MEDIUM • PYQ · CSIR NET JULY 2026

(A) $l = 1, m = -1$ for $\alpha = \beta$
(B) $l = 0, m = 0$ for $\alpha < \frac{\beta}{2}$
(C) $l = 1, m = -1$ for $\alpha < \frac{\beta}{2}$
(D) $l = 1, m = +1$ for $\alpha < \frac{\beta}{2}$

Q56. A container has a gas at temperature T with its N_1 molecules in the ground state and N_2 molecules in the excited state of energy hv. Both the levels are non-degenerate and A and B are the Einstein coefficients for spontaneous and stimulated emissions, respectively, between the two levels. Then the spectral density $\rho(v, T)$ of radiation inside the container is proportional to:

OPTICS • LASERS • MEDIUM • PYQ · CSIR NET JULY 2026

(A) $\frac{A}{B}\left(\frac{N_2}{N_1 - N_2}\right)$
(B) $\frac{A}{B}\left(\frac{N_2}{N_1 + N_2}\right)$
(C) $\frac{N_2}{\left(\frac{A}{B}\right)N_1 - N_2}$
(D) $\frac{N_2}{\left(\frac{A}{B}\right)N_1 + N_2}$

Q57. Consider a particle of mass m in an infinite potential well with walls at $x=0$ and $x=2L$. The particle is perturbed by a weak perturbation $H'=V_0 L\delta(x-3L/2)$. The first-order correction to its first excited state energy is:

QUANTUM MECHANICS • PERTUBATION THEORY • MEDIUM • PYQ · CSIR NET JULY 2026

(A) $2V_0$

(B) V_0

(C) $\frac{V_0}{2}$

(D) 0

Q58. Transverse electric modes TE_{mn} propagate in a hollow straight infinite metallic waveguide having rectangular cross section of sides 3 cm and 2 cm. The frequency (in GHz) of the lowest degenerate mode is:
(Given: speed of light $=3\times10^{8}$ $\mathrm{m/s}$)

ELECTROMAGNETISM • WAVEGUIDES AND TRANSMISSION LINE • MEDIUM • PYQ · CSIR NET JULY 2026

(A) 5.0

(B) 7.5

(C) 9.0

(D) 15.0

Q59. Assume a spherically uniform charge distribution in a nucleus having mass number A and radius $R=R_0A^{1/3}$ with $R_0\approx1.5\mathrm{fm}$. If the difference in electrostatic energy between two mirror nuclei, ${}_{z}^{A}X$ and ${}_{z-1}^{A}Y$, is 3.53 MeV, then the value of atomic number Z is closest to:
(Given: $\frac{e^2}{4\pi\epsilon_0}\approx1.44\mathrm{MeV.fm}$ and $1\mathrm{fm}=10^{-15}$ m)

NUCLEAR AND PARTICLE PHYSICS • NUCLEAR MODELS • MEDIUM • PYQ · CSIR NET JULY 2026

(A) 6

(B) 8

(C) 10

(D) 12

Q60. In a superconductor, the current density $\vec{J}$ and magnetic field $\vec{B}$ are related by the equation:
(Here, n is the number density of electrons, e is electron charge, m_e is electron mass and μ_0 is permeability of vacuum.)

CONDENSED MATTER PHYSICS • SUPERCONDUCTIVITY • MEDIUM • PYQ · CSIR NET JULY 2026

(A) $\frac{\partial}{\partial t}\left(\vec{\nabla}\times\vec{\jmath}-\frac{ne^2}{m_e}\vec{B}\right)=0$

(B) $\left(\vec{\nabla}\times\vec{\jmath}+\frac{ne^2}{m_e}\vec{B}\right)=0$

(C) $\left(\nabla^2\vec{B}-\mu_0\vec{\nabla}\times\vec{\jmath}\right)=0$

(D) $\left(\nabla^2\vec{\jmath}-\mu_0\vec{\nabla}\times\vec{B}\right)=0$

Q61. Consider a complex analytic function $f(x, y) = u(x, y) + iv(x, y)$. If $u(x, y) = x^3 - 3xy^2$, then $v(x, y)$ is:

MATHEMATICAL PHYSICS • COMPLEX ANALYSIS • MEDIUM • PYQ · CSIR NET JULY 2026

(A) $y^3 - 3xy^2$

(B) $3xy^2 - y^3$

(C) $3x^2y - y^3$

(D) $y^3 + 3xy^2$

Q62. An inertial observer S finds the electric field due to a large rectangular parallel plate capacitor to be directed along the y-axis. For another observer, moving at a relativistic speed with respect to S along the positive x-axis, which of the following statements is correct for the electric field:

ELECTROMAGNETISM • RELATIVISTIC EMT • MEDIUM • PYQ · CSIR NET JULY 2026

(A) Direction remains same but magnitude increases

(B) Direction remains same but magnitude decreases

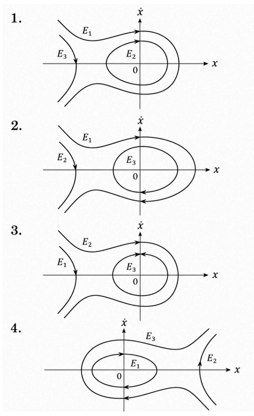
(C) Direction and magnitude both remain the same

(D) Direction tilts towards x-axis but the magnitude remains same

Q63. A particle is moving in one dimension in the potential

$$V(x) = \frac{1}{2}x^2 + \frac{1}{3}x^3.$$

The schematic phase-space diagrams for the total energies $E_1 > E_2 > E_3$ are best represented by:



CLASSICAL MECHANICS • PHASE SPACE • MEDIUM • PYQ · CSIR NET JULY 2026

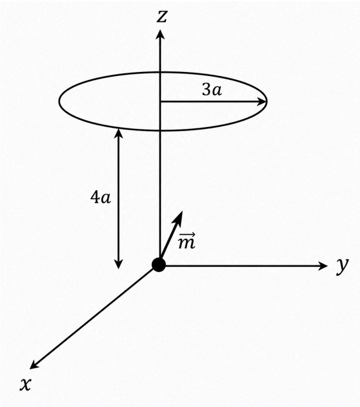
(A) 1

(B) 2

(C) 3

(D) 4

Q64. A magnetic dipole $\vec{m}$ is kept at the origin with its direction along $+z$-axis. At time $t=0$ it starts spinning about x-axis with angular speed ω. A wire loop of radius $3a$ is kept with its plane parallel to xy-plane and centered at $z=4a$ as shown in the figure. The emf E induced in the loop for $t>0$ is:
(Note that the magnetic flux through the given loop will be the same as through an appropriate section of a spherical shell of radius $5a$ with its center at the origin.)



ELECTROMAGNETISM • ELECTROMAGNETIC INDUCTION • MEDIUM • PYQ · CSIR NET JULY 2026

(A) $\left(\frac{9\mu_0 m\omega}{500a}\right)\sin\omega t$
(B) $\left(\frac{9\mu_0 m\omega}{500a}\right)\cos\omega t$
(C) $\left(\frac{9\mu_0 m\omega}{250a}\right)\sin\omega t$
(D) $\left(\frac{9\mu_0 m\omega}{250a}\right)\cos\omega t$

Q65. In four dimensions, a rank four tensor is given by T_{abcd}. It is symmetric under interchange of the indices a, b and interchange of the indices c, d. Moreover, the tensor is anti-symmetric under the interchange of pair of indices ab and cd. The number of independent components of the tensor is:

MATHEMATICAL PHYSICS • TENSORS • MEDIUM • PYQ · CSIR NET JULY 2026

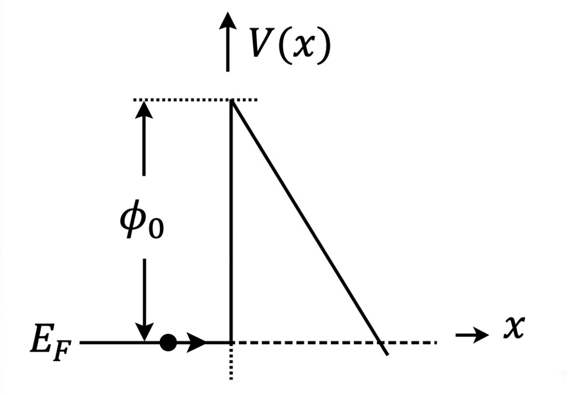
(A) 100
(B) 45
(C) 36
(D) 21

Q66. Newton-Raphson method is used for finding the roots of the polynomial $\left(x^2-1\right)(x-2)$. If the initial trial solutions are chosen as 0 and 0.5 , the algorithm converges to p and q, respectively. Then, which of the following is correct?

MATHEMATICAL PHYSICS • NUMERICAL METHODS • MEDIUM • PYQ · CSIR NET JULY 2026

(A) $p=1, q=-1$
(B) $p=-1, q=1$
(C) $p=2, q=1$
(D) $p=1, q=2$

Q67. The surface of a metal of work function ϕ_0 is subjected to an electric field $\mathcal{E}$. As a result, the electric potential profile just outside the surface is given by $V(x) = \phi_0 - x\mathcal{E}$ as shown in the figure. For electric field $\mathcal{E}$, the probability $p(\mathcal{E})$ of tunneling of an electron moving at the Fermi energy (E_F) can be found using the WKB approximation. If $\lambda = \sqrt{\frac{2me\phi_0^3}{\hbar^2}}$, then the value of the ratio $\frac{p(\varepsilon=2\lambda)}{p(\varepsilon=\lambda)}$ will be:
(Here, e and m are electron's charge and mass, respectively)



QUANTUM MECHANICS • WKB APPROXIMATION • MEDIUM • PYQ · CSIR NET JULY 2026

(A) 1.36
(B) 2.73
(C) 1.94
(D) 5.13

Q68. A dynamical system is described by a differential equation $\ddot{x} - \dot{x} + x^2 - 2x = 0$. The nature of the fixed point at the origin is:

CLASSICAL MECHANICS • SMALL OSCILLATIONS • MEDIUM • PYQ · CSIR NET JULY 2026

(A) unstable spiral
(B) stable node
(C) unstable hyperbola
(D) stable spiral

Q69. Consider the following decay processes of ρ^0 :

(A) $\rho^0 \to \pi^+ + \pi^-$
(B) $\rho^0 \to \pi^0 + \pi^0$

For the conservation of angular momentum and parity, which of the following options is correct?

NUCLEAR AND PARTICLE PHYSICS • PARTICLE PHYSICS • MEDIUM • PYQ · CSIR NET JULY 2026

(A) Both processes (A) and (B) are allowed
(B) Both processes (A) and (B) are forbidden
(C) Only process (A) is allowed
(D) Only process (B) is allowed

Q70. A biased random walker confined to the x-axis moves, at each step, with probability 0.6 to the right and with probability 0.4 to the left. If the walker takes 1000 steps starting from the origin, the probability that the walker is found exactly 200 steps to the right of the origin is close to:

STATISTICAL MECHANICS • RANDOM WALK AND DIFFUSION • MEDIUM • PYQ · CSIR NET JULY 2026

(A) 0.026
(B) 0.125
(C) 0.250
(D) 0.375

Q71. Potassium chloride (KCl) crystallizes in NaCl structure. The atomic form factors of K^{+} and Cl^{-} are the same. Then the first four X-ray diffraction peaks for KCl are:

CONDENSED MATTER PHYSICS • XRAY DIFFRACTION • MEDIUM • PYQ · CSIR NET JULY 2026

(A) (100), (110), (111), (200)
(B) (110),(200),(222),(310)
(C) (111), (200), (220), (311)
(D) (200),(220),(222),(400)

Q72. The table shows the g-factors of proton (p) and neutron (n) for orbital (g_l) and spin (g_s) angular momentum.

	g_l	g_s
p	1	5.58
n	0	-3.82

The magnetic moment of ${}_{8}^{17}\mathrm{O}$ nucleus, according to the single particle shell model, in units of nuclear magneton (μ_N) is:

NUCLEAR AND PARTICLE PHYSICS • NUCLEAR MODELS • MEDIUM • PYQ · CSIR NET JULY 2026

(A) 0.55
(B) 4.79
(C) -1.91
(D) -3.82

Q73. Resistivity of a monovalent metal, having atomic density 4×10^{22} atoms $/\mathrm{cm}^3$, is found to be $1 \mu\Omega$-cm. The approximate value of the mean free path of electrons in this metal is:
(Given: $\hbar = 10^{-34}$ J-s and $e = 1.6 \times 10^{-19}\mathrm{C}$)

CONDENSED MATTER PHYSICS • FREE ELECTRON THEORY • MEDIUM • PYQ · CSIR NET JULY 2026

(A) 1 nm
(B) 100 nm
(C) 10^4 nm
(D) 10^6 nm

Q74. A particle of mass m with momentum $\hbar\vec{k}$ scatters elastically to momentum $\hbar(\vec{k}+\vec{q})$ from a spherical potential

$$V(\vec{r}) = \begin{cases} V_0 & \text{for} \quad r \leq R \\ 0 & \text{for} \quad r > R \end{cases}.$$

Here, $q^2 = 4k^2 \sin^2 \frac{\theta}{2}$ with θ being the scattering angle. Within the firstorder Born approximation and for $qR \ll 1$, the differential scattering crosssection to the leading order in R is proportional to:

QUANTUM MECHANICS • SCATTERING THEORY • MEDIUM • PYQ · CSIR NET JULY 2026

(A) R^2

(B) R^4

(C) R^6

(D) R^8

Q75. Given is a set of 5 data points of the form (x_i, y_i, σ_i) with σ_i as the estimated uncertainty in y_i. This data is fitted to the form $y = ax + b$ and the minimized χ^2 (chi-square) is found to be 5.1. The reduced χ^2 for this fitting is:

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(A) 1.0

(B) 1.2

(C) 1.5

(D) 1.7

Answer Key

Q1: A	**Q2:** A	**Q3:** A	**Q4:** D	**Q5:** C	**Q6:** D
Q7: C	**Q8:** B	**Q9:** C	**Q10:** B	**Q11:** C	**Q12:** D
Q13: C	**Q14:** C	**Q15:** C	**Q16:** C	**Q17:** B	**Q18:** B
Q19: C	**Q20:** A	**Q21:** A	**Q22:** A, D	**Q23:** D	**Q24:** C
Q25: A	**Q26:** B	**Q27:** B	**Q28:** B	**Q29:** C	**Q30:** D
Q31: A	**Q32:** C	**Q33:** C	**Q34:** A	**Q35:** A	**Q36:** D
Q37: A	**Q38:** A	**Q39:** C	**Q40:** C	**Q41:** C	**Q42:** A
Q43: C	**Q44:** B	**Q45:** A	**Q46:** A	**Q47:** B	**Q48:** A
Q49: A	**Q50:** C	**Q51:** B	**Q52:** B	**Q53:** Dropped — full marks	**Q54:** D
Q55: C	**Q56:** A	**Q57:** B	**Q58:** D	**Q59:** B	**Q60:** B
Q61: C	**Q62:** A	**Q63:** A	**Q64:** C	**Q65:** B	**Q66:** C
Q67: C	**Q68:** C	**Q69:** C	**Q70:** A	**Q71:** D	**Q72:** C
Q73: B	**Q74:** C	**Q75:** A			

Solutions

Q1.

The bells ring together again after a time equal to the LCM of their individual intervals.

$$30 = 2 \times 3 \times 5, \quad 40 = 2^3 \times 5, \quad 45 = 3^2 \times 5$$

$$\text{LCM} = 2^3 \times 3^2 \times 5 = 360 \text{ sec}$$

$$360 \text{ sec} = 6 \text{ min}$$

$$\text{Starting at 4:00 am, they will next ring together at } 4:00 + 0:06 = \text{4:06 am}$$

Correct answer: Option 1, 4:06 am

Q2.

Each given number is a perfect cube, and the code maps it to another perfect cube.

$$1000 = 10^3 \rightarrow 1728 = 12^3 \qquad \text{(base increases by 2)}$$

$$125 = 5^3 \rightarrow 343 = 7^3 \qquad \text{(base increases by 2)}$$

$$343 = 7^3 \rightarrow 729 = 9^3 \qquad \text{(base increases by 2)}$$

$$\text{So the coding rule is } n^3 \rightarrow (n+2)^3$$

$$512 = 8^3 \rightarrow (8+2)^3 = 10^3 = 1000$$

Correct answer: Option 1, 1000

Q3.

$$\text{Total employees} = 2000$$

$$\text{Admin employees} = 20\% \text{ of } 2000 = 400$$

$$\text{Male : Female in Admin} = 2:3$$

$$\text{Female employees in Admin} = \frac{3}{5} \times 400 = 240$$

$$\text{Non teaching employees} = 28\% \text{ of } 2000 = 560$$

$$\text{Male : Female in Non teaching} = 3:4$$

$$\text{Female employees in Non teaching} = \frac{4}{7} \times 560 = 320$$

$$\text{Required ratio} = 240:320 = 3:4$$

Correct answer: Option 1, 3 : 4

Q4.

$$\text{The GDP contribution is only from the working population, distributed across Service, Industry, Agriculture and Others as shown in the chart.}$$

$$\text{Among the working sectors: Service} = 38\%,\ \text{Industry} = 28\%,\ \text{Agriculture} = 10\%,\ \text{Others} = 24\%$$

$$\text{The non working population, by definition, does not participate in any economic activity and hence contributes } 0\% \text{ to the GDP.}$$

$$\text{Since } 0\% < 10\%,\ \text{the non working population has the lowest contribution.}$$

Correct answer: Option 4, Non working population

Q5.

$$\text{Assume total candidates} = 100$$

$$\text{Men} = 60, \quad \text{Women} = 40$$

$$\text{Men clearing qualifying test} = 70\% \text{ of } 60 = 42$$
$$\text{Men successful in final test} = 80\% \text{ of } 42 = 33.6$$

$$\text{Women clearing qualifying test} = 75\% \text{ of } 40 = 30$$
$$\text{Women successful in final test} = 70\% \text{ of } 30 = 21$$

$$\text{Overall success rate} = \frac{33.6 + 21}{100} \times 100\% = 54.6\%$$
$$\text{This is above } 50\%,\ \text{so statement 1 is false}$$

$$\text{Success rate for men} = \frac{33.6}{60} \times 100\% = 56\%$$
$$\text{Success rate for women} = \frac{21}{40} \times 100\% = 52.5\%$$
$$\text{Men have a higher success rate, so statement 2 is false}$$

$$\text{Number of men cleared} = 33.6 > \text{Number of women cleared} = 21$$
$$\text{so statement 3 is true}$$

Correct answer: Option 3, More men cleared the examination than women

Q6.

$$\text{Let the four positive integers in sorted order be } A \le B \le C \le D.$$

$$\text{Median} = \frac{B+C}{2} = 7 \;\Rightarrow\; B+C = 14$$

$$\text{For the mode to be uniquely 6, the value 6 must occur at least twice}$$
$$\text{and more often than any other value in the list.}$$

$$\text{To keep } D \text{ as small as possible, take } B = 6, \text{ so } C = 14 - 6 = 8$$

$$\text{Now 6 appears only once (as } B\text{), so set } A = 6 \text{ as well, giving two 6's}$$

$$\text{Sequence so far: } A, B, C, D = 6, 6, 8, D, \quad \text{with } D \ge C = 8$$

$$\text{If } D = 8, \text{ the list becomes } 6, 6, 8, 8, \text{ giving two modes (6 and 8), violating uniqueness}$$

$$\text{So } D \text{ must be strictly greater than } 8, \text{ i.e. } D \ge 9$$

$$\text{Taking } D = 9: \text{ list} = 6, 6, 8, 9$$
$$\text{Mode} = 6 \text{ (unique, appears twice)}, \quad \text{Median} = \frac{6+8}{2} = 7 \checkmark$$

$$\text{Both conditions are satisfied, and no smaller } D \text{ works.}$$

Correct answer: Option 4, D = 9

Q7.

$$\text{Let the principal be } P. \text{ The sum doubles in 5 years, so the interest earned}$$
$$\text{in 5 years equals } P.$$

$$\text{Using } SI = \frac{PRT}{100}, \quad P = \frac{P \times R \times 5}{100}$$
$$\Rightarrow R = 20\% \text{ per annum}$$

$$\text{For the sum to become 16 times itself, the interest required is}$$
$$16P - P = 15P$$

$$15P = \frac{P \times 20 \times T}{100}$$
$$\Rightarrow T = \frac{15 \times 100}{20} = 75 \text{ years}$$

Correct answer: Option 3, 75

Q8.

Consider the three letters of each term separately, converting to their position in the alphabet (A=1, B=2, $\ldots$, Z=26).

$$\text{First letters: } X, T, P, ?, H \;\rightarrow\; 24, 20, 16, ?, 8$$
$$\text{Each term decreases by 4: } 24 \rightarrow 20 \rightarrow 16 \rightarrow 12 \rightarrow 8$$
$$\Rightarrow ? = 12 = L$$

$$\text{Second letters: } U, Q, M, ?, E \;\rightarrow\; 21, 17, 13, ?, 5$$
$$\text{Each term decreases by 4: } 21 \rightarrow 17 \rightarrow 13 \rightarrow 9 \rightarrow 5$$
$$\Rightarrow ? = 9 = I$$

$$\text{Third letters: } R, N, J, ?, B \;\rightarrow\; 18, 14, 10, ?, 2$$
$$\text{Each term decreases by 4: } 18 \rightarrow 14 \rightarrow 10 \rightarrow 6 \rightarrow 2$$
$$\Rightarrow ? = 6 = F$$

$$\text{Combining the three letters gives the missing term} = LIF$$

Correct answer: Option 2, LIF

Q9.

In species interactions, the effect on each partner can be classified as beneficial (+), harmful (-), or neutral (0).

Commensalism: one species benefits (+), the other is unaffected (0)

Amensalism: one species is harmed (-), the other is unaffected (0)

Competition: both species are harmed (-, -)

Parasitism: the parasite benefits (+), the host is harmed (-)

Predation: the predator benefits (+), the prey is harmed (-)

The condition of one species benefiting while the other is harmed corresponds exactly to Parasitism and Predation.

Correct answer: Option 3, Parasitism and Predation

Q10.

Let the two natural numbers be x and y, with $x > y$.

$$x - y = 6$$

$$x^2 - y^2 = 60 \;\Rightarrow\; (x-y)(x+y) = 60$$

$$6(x+y) = 60 \;\Rightarrow\; x + y = 10$$

Solving the two equations:

$$x - y = 6, \quad x + y = 10$$

$$\Rightarrow\; x = 8, \quad y = 2$$

$$\text{Sum of cubes} = x^3 + y^3 = 8^3 + 2^3 = 512 + 8 = 520$$

Correct answer: Option 2, 520

Q11.

Let P be at the origin: $P = (0, 0)$

Q is South-West of P: $Q = (-1, -1)$

R is East of Q and South-East of P: $R = (1, -1)$
(x-coordinate greater than Q's, and positive x, negative y relative to P)

The line joining Q and P has direction $(1, 1)$, i.e. the line $y = x$

T is North of R (same x-coordinate as R, larger y-coordinate) and lies on this line $y = x$

$$\Rightarrow\; T = (1, 1)$$

Direction of T from P: since $T = (1, 1)$ lies along the positive 45° diagonal from P, T is located to the North-East of P.

Correct answer: Option 3, North-East

Q12.

Entropy is a measure of disorder or randomness in a system. Processes such as melting, boiling, or dissolving increase the freedom of motion of molecules and hence increase entropy.

Water is boiled: liquid $\to$ gas, molecules gain much greater freedom $\Rightarrow\ \Delta S > 0$

Sugar is dissolved in water: solid lattice $\to$ ions/molecules dispersed randomly in solution $\Rightarrow\ \Delta S > 0$

Ice is heated to melt: solid $\to$ liquid, molecular disorder increases $\Rightarrow\ \Delta S > 0$

Egg is boiled in water: on heating, the albumin (protein) present in egg white denatures and coagulates, converting from a loosely folded, mobile liquid form into a cross-linked, rigid solid network $\Rightarrow$ molecular disorder decreases $\Rightarrow\ \Delta S < 0$

Correct answer: Option 4, Egg is boiled in water

Q13.

Chromatography is a general technique used to separate the individual components of a mixture based on differences in their movement through a stationary phase under the influence of a mobile phase.

Distillation separates substances specifically based on differences in boiling points.

Filtration separates an insoluble solid from a liquid based on particle size.

Crystallisation separates a solid solute from a solution by forming crystals on cooling or evaporation.

Among these, chromatography is the most general process specifically used to separate the substances present in a chemical mixture, distinguishing components based on their differing affinities and mobilities.

Correct answer: Option 3, Chromatography

Q14.

With a balance scale, each weighing has 3 possible outcomes: left heavier, right heavier, or balanced. With n weighings, up to 3^n coins can be distinguished.

Divide the 9 coins into 3 groups of 3 coins each.

Weighing 1: Place any two groups (3 coins vs 3 coins) on the scale.
If balanced, the counterfeit is in the third (unweighed) group.
If unbalanced, the counterfeit is in the lighter group.
This identifies the group of 3 containing the counterfeit coin.

Weighing 2: Take the identified group of 3 coins, place any two of them on the scale (1 vs 1).
If balanced, the counterfeit is the third, unweighed coin.
If unbalanced, the counterfeit is the lighter one.

Thus the counterfeit coin is isolated in just 2 weighings, consistent with $3^2 = 9$ being exactly the number of coins.

Correct answer: Option 3, 2

Q15.

$$\text{Diameter} = 14 \text{ cm} \;\Rightarrow\; \text{radius, } r = 7 \text{ cm}$$

$$\text{Area of semicircle} = \frac{1}{2}\pi r^2 = \frac{1}{2} \times \frac{22}{7} \times 7^2 = 77 \text{ sq. cm}$$

The largest triangle inscribed in a semicircle has its base equal to the diameter, and its apex on the semicircular arc, with height equal to the radius (this gives maximum height, hence maximum area).

$$\text{Base of triangle} = 14 \text{ cm}, \quad \text{Height of triangle} = r = 7 \text{ cm}$$

$$\text{Area of triangle} = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 14 \times 7 = 49 \text{ sq. cm}$$

$$\text{Area not occupied by the triangle} = 77 - 49 = 28 \text{ sq. cm}$$

Correct answer: Option 3, 28 sq. cm

Q16. Sales figures (in crores): $2020 = 400,\ 2021 = 600,\ 2022 = 1000,\ 2023 = 800,\ 2024 = 1200,\ 2025 = 920$

$$\text{Percentage increase} = \frac{\text{Sales(current year)} - \text{Sales(previous year)}}{\text{Sales(previous year)}} \times 100\%$$

$$2021 \text{ vs } 2020: \ \frac{600-400}{400} \times 100\% = 50\%$$
$$2022 \text{ vs } 2021: \ \frac{1000-600}{600} \times 100\% = 66.67\%$$
$$2023 \text{ vs } 2022: \ \text{Sales decreased, not an increase}$$
$$2024 \text{ vs } 2023: \ \frac{1200-800}{800} \times 100\% = 50\%$$
$$2025 \text{ vs } 2024: \ \text{Sales decreased, not an increase}$$

$$\text{Comparing the increases: } 50\%,\ 66.67\%,\ 50\%$$
$$\text{The maximum percentage increase occurs in 2022}$$

Correct answer: Option 3, 2022

Q17.

$$\text{Given: } P+Q \Rightarrow P \text{ is brother of } Q; \quad P \times Q \Rightarrow P \text{ is father of } Q; \quad P-Q \Rightarrow P \text{ is sister of } Q$$

$$\text{Check option 2: } K+Y \times I - Z$$

$$K+Y \ \Rightarrow \ K \text{ is brother of } Y$$
$$Y \times I \ \Rightarrow \ Y \text{ is father of } I$$
$$I - Z \ \Rightarrow \ I \text{ is sister of } Z \ \Rightarrow \ I \text{ is female}$$

$$\text{Since } Y \text{ is the father of } I, \text{ and } K \text{ is the brother of } Y,$$
$$K \text{ is the uncle of } I.$$

$$\text{Since } I \text{ is female (established as sister of Z), } I \text{ is the niece of } K.$$

Correct answer: Option 2, K + Y × I − Z

Q18.

$$\text{Let the two numbers be } 5x \text{ and } 3x, \text{ consistent with the given ratio } 5:3$$

$$\text{Difference} = 5x - 3x = 2x = 10$$
$$\Rightarrow \ x = 5$$

$$\text{The two numbers are: } 5x = 25 \text{ and } 3x = 15$$

$$\text{Product} = 25 \times 15 = 375$$

Correct answer: Option 2, 375

Q19.

$$\text{Let present males} = M,\ \text{present females} = F$$

$$M + F = 6000$$

$$\text{After increase: } 1.14M + 1.22F = 6900$$

$$\text{Substituting } M = 6000 - F:$$
$$1.14(6000 - F) + 1.22F = 6900$$

$$6840 - 1.14F + 1.22F = 6900$$

$$6840 + 0.08F = 6900$$

$$0.08F = 60$$

$$F = 750$$

Correct answer: Option 3, 750

Q20.

In fig (X), moving through the cells of the 2×2 grid, the diagonal line and the shaded half of the circle both rotate by a fixed pattern from one cell to the next.

Tracking the direction of the diagonal (bottom-left to top-right, alternating with top-left to bottom-right) and the position of the shaded half of the circle (rotating consistently around the circle), the missing fourth cell must continue this same rotational sequence.

Matching both the diagonal orientation and the shading position that completes the rotation, the figure with the diagonal from bottom-left to top-right and the left half of the circle shaded black fits correctly into the pattern.

Correct answer: Option 1

Q21.

$$\text{The 3-bit output ABC represents the battery voltage in steps of } 0.5\text{ V},$$
$$\text{so Voltage} = (\text{decimal value of ABC}) \times 0.5\text{ V}$$

$$\text{We require output HIGH when voltage} > 2\text{ V}, \text{ i.e. decimal value} > 4$$

$$\text{Since values are integers, this means decimal value} \geq 5, \text{ i.e. ABC} = 101, 110, 111$$

$$\text{Writing the sum of minterms:}$$
$$f(A,B,C) = A\bar{B}C + AB\bar{C} + ABC$$

$$\text{Grouping the last two terms: } AB\bar{C} + ABC = AB(\bar{C} + C) = AB$$

$$\Rightarrow \; f = A\bar{B}C + AB$$

$$\text{Using the absorption identity } \bar{B}C + B = B + C:$$
$$f = A(\bar{B}C + B) = A(B + C) = AB + AC$$

Correct answer: Option 1, AB + AC

Q22.

Grounded plane at $z = 0,$ point charge q at $(0, 0, a_0)$. By the method of images, an image charge $-q$ at $(0, 0, -a_0)$ satisfies $V = 0$ on the plane $z = 0$.

$$\text{Potential for } z > 0:$$

$$V(\vec{r}) = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{|\vec{r} - a_0\hat{k}|} - \frac{1}{|\vec{r} + a_0\hat{k}|}\right]$$

$$\text{On the z-axis:} \quad V(z) = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{|z - a_0|} - \frac{1}{z + a_0}\right]$$

$$\text{Given } \frac{q}{4\pi\epsilon_0 a_0} = 24\text{ V}:$$

$$V(z) = 24a_0\left[\frac{1}{|z - a_0|} - \frac{1}{z + a_0}\right]$$

$$\text{At } z = 2a_0: \quad V = 24a_0\left[\frac{1}{a_0} - \frac{1}{3a_0}\right] = 24a_0 \times \frac{2}{3a_0} = 16\text{ Volt}$$

$$\text{At } z = 3a_0: \quad V = 24a_0\left[\frac{1}{2a_0} - \frac{1}{4a_0}\right] = 6\text{ Volt} \neq 8\text{ Volt}$$

$$\text{At } z = 4a_0: \quad V = 24a_0\left[\frac{1}{3a_0} - \frac{1}{5a_0}\right] = 3.2\text{ Volt} \neq 4\text{ Volt}$$

$$\text{At } z = a_0/2: \quad V = 24a_0\left[\frac{2}{a_0} - \frac{2}{3a_0}\right] = 24a_0 \times \frac{4}{3a_0} = 32\text{ Volt}$$

Both $z = 2a_0$ and $z = a_0/2$ satisfy the formula exactly, while the other two options do not match. This means the question, as posed with a single correct choice, is flawed since two options are simultaneously valid.

Correct answer: Options 1 and 4 both satisfy the potential formula exactly (16 V at $z = 2a_0$ and 32 V at $z = a_0/2$); the question is ambiguous as a single-answer MCQ.

Q23.

For any time-independent observable $O,$ Ehrenfest's theorem gives

$$\frac{d\langle O\rangle}{dt} = \frac{i}{\hbar}\langle\phi(t)|[H,O]|\phi(t)\rangle$$

(A) If $[O,H]=0,$ this commutator vanishes identically as an operator, so $\langle\phi(t)|[H,O]|\phi(t)\rangle = 0$ at every instant, for every state

$$\Rightarrow\ \frac{d\langle O\rangle}{dt} = 0 \text{ always } \Rightarrow\ \text{(A) TRUE}$$

(B) If $|\phi(t)\rangle$ is an eigenstate of H with energy $E,$ it evolves only by an overall phase: $|\phi(t)\rangle = e^{-iEt/\hbar}|\phi(0)\rangle,$ and this phase cancels in $\langle\phi(t)|O|\phi(t)\rangle,$ giving a constant value $\Rightarrow$ (B) TRUE

(C) If $|\phi(t)\rangle$ is an eigenstate of O at every instant, then $\langle O(t)\rangle = \lambda(t),$ the corresponding eigenvalue of O at that instant.

Since O is Hermitian with a fixed, discrete, non-degenerate spectrum, and the state $|\phi(t)\rangle$ evolves continuously in time (unitary evolution generated by H is continuous), the function $\lambda(t)$ must also vary continuously.

A continuous function that only ever takes values in a discrete set must be constant $\Rightarrow$ $\lambda(t) = \lambda(0)$ for all t $\Rightarrow$ (C) TRUE

(D) $[O,H]$ is anti-Hermitian (since O, H are Hermitian), so its eigenvalues are purely imaginary: $[O,H]|\phi(t)\rangle = i\lambda\,|\phi(t)\rangle,$ λ real, and by the same continuity argument λ is constant in time.

$$\frac{d\langle O\rangle}{dt} = \frac{i}{\hbar}\langle\phi(t)|[H,O]|\phi(t)\rangle = \frac{i}{\hbar}(-i\lambda) = \frac{\lambda}{\hbar}$$

This is a fixed nonzero rate (unless $\lambda = 0$), so $\langle O(t)\rangle$ grows linearly with time rather than staying constant $\Rightarrow$ (D) FALSE

Correct answer: Option 4, Only (A), (B) and (C) are correct

Q24.

The circuit consists of two complementary MOSFETs connected in series between $+5$ V and ground, with their gates tied together to the input and their drains tied together to the output.

The upper transistor (source at $+5$ V, arrow pointing into the gate) is a p-channel MOSFET (PMOS).

The lower transistor (source at ground, arrow pointing out of the gate) is an n-channel MOSFET (NMOS).

This series combination of a PMOS (pull-up) and an NMOS (pull-down) sharing a common gate and common drain is the standard building block of CMOS (Complementary MOS) logic.

$$\text{When in} = 0: \ \text{PMOS is ON, NMOS is OFF} \ \Rightarrow \ \text{out is pulled to } +5 \text{ V } (1)$$
$$\text{When in} = 1: \ \text{PMOS is OFF, NMOS is ON} \ \Rightarrow \ \text{out is pulled to ground } (0)$$

This input-output behavior ($\text{out} = \overline{\text{in}}$) is exactly that of an inverter, implemented using CMOS technology.

Correct answer: Option 3, CMOS NOT Gate

Q25.

Ground state of the original well (width a):

$$\psi_i(x) = \sqrt{\frac{2}{a}}\sin\left(\frac{\pi x}{a}\right), \quad 0 < x < a, \qquad E_1 = \frac{\pi^2\hbar^2}{2ma^2}$$

Adiabatic process:

By the adiabatic theorem, the particle remains in the instantaneous ground state (same quantum number) as the wall slowly moves to $2a$.

$$\langle E\rangle_a = \frac{\pi^2\hbar^2}{2m(2a)^2} = \frac{E_1}{4}$$

Sudden process:

The wavefunction has no time to change; immediately after the wall jumps to $2a$, the state is still $\psi_i(x)$ on $(0,a)$ and zero on $(a,2a)$.

The expectation value of energy in the new well is computed using

$$\langle E\rangle_s = \frac{\hbar^2}{2m}\int_0^{2a} |\psi_i'(x)|^2\,dx$$

$$\psi_i'(x) = \sqrt{\frac{2}{a}}\,\frac{\pi}{a}\cos\left(\frac{\pi x}{a}\right), \quad 0 < x < a$$

$$\int_0^{a} |\psi_i'(x)|^2\,dx = \frac{2}{a}\cdot\frac{\pi^2}{a^2}\int_0^{a}\cos^2\left(\frac{\pi x}{a}\right)dx = \frac{2\pi^2}{a^3}\cdot\frac{a}{2} = \frac{\pi^2}{a^2}$$

$$\langle E\rangle_s = \frac{\hbar^2}{2m}\cdot\frac{\pi^2}{a^2} = \frac{\pi^2\hbar^2}{2ma^2} = E_1$$

Taking the ratio:

$$\frac{\langle E\rangle_s}{\langle E\rangle_a} = \frac{E_1}{E_1/4} = 4$$

Correct answer: Option 1, 4:1

Q26.

$$\text{Given: } [\hat{R},[\hat{R},\hat{S}]]=0,\ \text{i.e. } \hat{R} \text{ commutes with the commutator } [\hat{R},\hat{S}]$$

$$\text{First establish the identity for integer powers of } \hat{R},\ \text{using}$$
$$[\hat{R}^n,\hat{S}]=\hat{R}[\hat{R}^{n-1},\hat{S}]+[\hat{R},\hat{S}]\hat{R}^{n-1}$$

$$\text{Since } [\hat{R},\hat{S}] \text{ commutes with } \hat{R},\ \text{it also commutes with any power of } \hat{R},$$
$$\text{so } [\hat{R},\hat{S}]\hat{R}^{n-1}=\hat{R}^{n-1}[\hat{R},\hat{S}]$$

$$\text{By induction, this gives the standard result:}$$
$$[\hat{R}^n,\hat{S}]=n\hat{R}^{n-1}[\hat{R},\hat{S}]$$

$$\text{Now expand } e^{\alpha\hat{R}} \text{ as a power series:}$$
$$e^{\alpha\hat{R}}=\sum_{n=0}^{\infty}\frac{\alpha^n\hat{R}^n}{n!}$$
$$\left[e^{\alpha\hat{R}},\hat{S}\right]=\sum_{n=0}^{\infty}\frac{\alpha^n}{n!}[\hat{R}^n,\hat{S}]=\sum_{n=1}^{\infty}\frac{\alpha^n}{n!}\cdot n\hat{R}^{n-1}[\hat{R},\hat{S}]$$
$$=\sum_{n=1}^{\infty}\frac{\alpha^n}{(n-1)!}\hat{R}^{n-1}[\hat{R},\hat{S}]=\alpha[\hat{R},\hat{S}]\sum_{m=0}^{\infty}\frac{\alpha^m\hat{R}^m}{m!}$$
$$=\alpha e^{\alpha\hat{R}}[\hat{R},\hat{S}]$$

Correct answer: Option 2, $\alpha e^{\alpha\hat{R}}[\hat{R},\hat{S}]$

Q27.

Decompose the incident linearly polarized wave into equal-amplitude right and left circularly polarized components:

$$\vec{E}_{\text{inc}} = E_0\,(\hat{e}_R + \hat{e}_L)$$

At normal incidence, each circular component reflects independently, governed by the Fresnel reflection coefficient appropriate to its own refractive index:

$$r_R = \frac{1-n_R}{1+n_R}, \qquad r_L = \frac{1-n_L}{1+n_L}$$

Since $n_R \neq n_L,$ we have $r_R \neq r_L,$ so the reflected wave contains unequal amplitudes of right and left circular polarization:

$$\vec{E}_{\text{ref}} = E_0\,(r_R\,\hat{e}_R + r_L\,\hat{e}_L)$$

A superposition of right and left circular polarizations with EQUAL amplitude gives linear polarization; with ONLY one component present gives pure circular polarization. Any other combination of unequal, nonzero amplitudes traces out an ellipse in general.

Since $r_R \neq r_L$ and both are generally nonzero, the reflected wave is a combination of unequal circular components, and hence traces an elliptical polarization state.

Correct answer: Option 2, Elliptical

Q28.

For $\{e_1, e_2, e_3, e_4\}$ to form a basis of the 4-dimensional space V_4, the determinant of the matrix formed by these four vectors as rows must be nonzero.

$$e_1 = (1,1,0,0), \quad e_2 = (0,1,1,0), \quad e_3 = (0,0,1,1)$$

Testing Option 2: $e_4 = (1,1,1,0)$

$$\begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$$

Row reduce: $R_4 \to R_4 - R_1 = (0,0,1,0)$

$$\begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$R_4 \to R_4 - R_3 = (0,0,0,-1)$$

$$\begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

This is upper triangular, so $\det = 1 \times 1 \times 1 \times (-1) = -1 \neq 0$

$\Rightarrow \ \{e_1, e_2, e_3, e_4\}$ are linearly independent and form a basis

Checking the other options quickly:

Option 1: $e_4 = (1,1,1,1) = e_1 + e_3 \ \Rightarrow$ linearly dependent

Option 3: $e_4 = (1,0,-1,0) = e_1 - e_2 \ \Rightarrow$ linearly dependent

Option 4: $e_4 = (0,1,1,0) = e_2 \ \Rightarrow$ trivially dependent

Correct answer: Option 2, $(1,1,1,0)$

Q29.

$$\text{Six distinguishable particles, each in state } -E_0, 0, \text{ or } +E_0.$$
$$\text{Total energy zero means: (particles at } +E_0) = (\text{particles at } -E_0) = k$$

$$\text{Remaining } (6-2k) \text{ particles sit at energy } 0, \quad k = 0, 1, 2, 3$$

$$\text{Number of ways to choose which particles go where:}$$
$$\Omega(k) = \binom{6}{k}\binom{6-k}{k}$$

$$k = 0: \ \binom{6}{0}\binom{6}{0} = 1$$
$$k = 1: \ \binom{6}{1}\binom{5}{1} = 6 \times 5 = 30$$
$$k = 2: \ \binom{6}{2}\binom{4}{2} = 15 \times 6 = 90$$
$$k = 3: \ \binom{6}{3}\binom{3}{3} = 20 \times 1 = 20$$

$$\text{Total} = 1 + 30 + 90 + 20 = 141$$

Correct answer: Option 3, 141

Q30.

Let x_1, x_2, x_3 be displacements of the three masses. The spring arrangement is: wall $- 2k - m_1 - k - m_2 - k - m_3 - 2k -$ wall

Equations of motion (Newton's second law for each mass):

$$m\ddot{x}_1 = -2kx_1 - k(x_1 - x_2) = -3kx_1 + kx_2$$
$$m\ddot{x}_2 = -k(x_2 - x_1) - k(x_2 - x_3) = kx_1 - 2kx_2 + kx_3$$
$$m\ddot{x}_3 = -k(x_3 - x_2) - 2kx_3 = kx_2 - 3kx_3$$

This gives the matrix equation $\ddot{\vec{x}} = -\frac{k}{m}M\vec{x},$ with

$$M = \begin{pmatrix} 3 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 3 \end{pmatrix}$$

The normal mode frequencies $\omega_1^2, \omega_2^2, \omega_3^2$ are the eigenvalues of $\frac{k}{m}M$.
A key property of eigenvalues is that their sum always equals the trace of the matrix, regardless of the individual eigenvectors:

$$\omega_1^2 + \omega_2^2 + \omega_3^2 = \frac{k}{m}\,\mathrm{Tr}(M) = \frac{k}{m}(3 + 2 + 3) = \frac{8k}{m}$$

Since $\omega_0^2 = \frac{k}{m},$ this gives

$$\omega_1^2 + \omega_2^2 + \omega_3^2 = 8\omega_0^2$$

Correct answer: Option 4, $8\omega_0^2$

Q31.

The spin operator projected along the radial direction $\hat{r}(\theta, \varphi)$ is

$$S_r = \vec{S} \cdot \hat{r} = \frac{\hbar}{2} \begin{pmatrix} \cos\theta & \sin\theta\, e^{-i\varphi} \\ \sin\theta\, e^{i\varphi} & -\cos\theta \end{pmatrix}$$

Its normalized eigenstate for eigenvalue $+\dfrac{\hbar}{2}$ is the standard result

$$|+\rangle = \begin{pmatrix} \cos\dfrac{\theta}{2} \\ e^{i\varphi} \sin\dfrac{\theta}{2} \end{pmatrix}$$

The probability of measuring $S_r = +\dfrac{\hbar}{2}$ on the given state $|\beta\rangle$ is

$$P(+\hbar/2) = |\langle + | \beta \rangle|^2$$

$$\langle + | \beta \rangle = \cos\frac{\theta}{2} \sin\frac{\theta}{2} + \left(e^{-i\varphi} \sin\frac{\theta}{2}\right) \left(e^{i\varphi} \cos\frac{\theta}{2}\right)$$

$$= \cos\frac{\theta}{2} \sin\frac{\theta}{2} + \sin\frac{\theta}{2} \cos\frac{\theta}{2} = 2 \sin\frac{\theta}{2} \cos\frac{\theta}{2} = \sin\theta$$

$$\Rightarrow \; P(+\hbar/2) = |\sin\theta|^2 = \sin^2\theta$$

Check: the orthogonal probability $P(-\hbar/2) = \cos^2\theta,$ and $\sin^2\theta + \cos^2\theta = 1,$ confirming normalization.

Correct answer: Option 1, $\sin^2\theta$

Q32.

The variation of melting point with pressure is governed by the Clausius-Clapeyron equation for the solid-liquid transition:

$$\frac{dP}{dT} = \frac{L}{T\,\Delta v}$$

where $\Delta v = v_{\text{liquid}} - v_{\text{solid}}$ is the specific volume change on melting, and L is the latent heat per unit mass.

$$v_{\text{water}} = \frac{1}{1} = 1\ \text{cm}^3/\text{g}, \qquad v_{\text{ice}} = \frac{1}{0.917} = 1.0905\ \text{cm}^3/\text{g}$$

$$\Delta v = 1 - 1.0905 = -0.0905\ \text{cm}^3/\text{g} = -9.05 \times 10^{-8}\ \text{m}^3/\text{g}$$

$$\frac{dT}{dP} = \frac{T\,\Delta v}{L} = \frac{273 \times (-9.05 \times 10^{-8})}{334} = -7.40 \times 10^{-8}\ \text{K}/\text{Pa}$$

Converting to K/atm ($1\ \text{atm} = 1.01325 \times 10^5\ \text{Pa}$):

$$\frac{dT}{dP} \approx -7.40 \times 10^{-8} \times 1.01325 \times 10^5 \approx -0.0075\ \text{K}/\text{atm}$$

This negative sign reflects the anomalous behavior of water/ice: since ice is less dense than water, increasing pressure lowers the melting point.

For a pressure increase of $\Delta P = 140 - 1 \approx 139$ atm :

$$\Delta T \approx -0.0075 \times 139 \approx -1.04\ \text{K}$$

$$\Rightarrow\ T_m \approx 273 - 1.04 \approx 272\ \text{K}$$

Correct answer: Option 3, At 140 atm pressure, $T_m \simeq 272$ K

Q33.

For a central force, the orbit equation in terms of $u = \frac{1}{r}$ is

$$\frac{d^2u}{d\theta^2} + u = -\frac{m}{L^2u^2}\,F\left(\frac{1}{u}\right)$$

Given orbit: $r = k\,e^{\alpha\theta} \;\Rightarrow\; u = \frac{1}{r} = \frac{1}{k}e^{-\alpha\theta}$

$$\frac{du}{d\theta} = -\alpha\,\frac{1}{k}e^{-\alpha\theta} = -\alpha u$$

$$\frac{d^2u}{d\theta^2} = -\alpha\frac{du}{d\theta} = -\alpha(-\alpha u) = \alpha^2 u$$

Substituting into the orbit equation:

$$\alpha^2 u + u = -\frac{m}{L^2u^2}F\left(\frac{1}{u}\right)$$

$$(1+\alpha^2)u = -\frac{m}{L^2u^2}F(r)$$

$$F(r) = -\frac{L^2}{m}(1+\alpha^2)\,u^3 = -\frac{L^2(1+\alpha^2)}{m}\cdot\frac{1}{r^3}$$

Since k, α, m, L are all constants, this shows

$$F(r) \propto \frac{1}{r^3}$$

Correct answer: Option 3, $\frac{1}{r^3}$

Q34.

$$L = x^2 + x\dot{x} + \frac{\dot{x}^2}{2}$$

$$\text{Generalized momentum:} \quad p = \frac{\partial L}{\partial \dot{x}} = x + \dot{x}$$

$$\Rightarrow \ \dot{x} = p - x$$

$$\text{Hamiltonian: } H = p\dot{x} - L$$

$$p\dot{x} = p(p-x) = p^2 - px$$

$$L = x^2 + x(p-x) + \frac{(p-x)^2}{2} = x^2 + xp - x^2 + \frac{p^2 - 2px + x^2}{2}$$

$$= xp + \frac{p^2}{2} - px + \frac{x^2}{2} = \frac{p^2}{2} + \frac{x^2}{2}$$

$$H = p^2 - px - \left(\frac{p^2}{2} + \frac{x^2}{2}\right) = \frac{p^2}{2} - px - \frac{x^2}{2}$$

$$H = \frac{p^2 - x^2 - 2px}{2}$$

Correct answer: Option 1, $\dfrac{p^2 - x^2 - 2xp}{2}$

Q35.

$$\text{Canonical momentum: } \vec{p} = m\vec{v} + q\vec{A}, \quad \oint_C \vec{p}\cdot d\vec{l} = \oint_C m\vec{v}\cdot d\vec{l} + q\oint_C \vec{A}\cdot d\vec{l}$$

Kinetic term:

Since $d\vec{l} = \vec{v}\,dt,$ we have $m\vec{v}\cdot d\vec{l} = mv^2\,dt,$ always positive.

$$\text{From centripetal balance, } qvB = \frac{mv^2}{R} \Rightarrow mv = qBR$$

$$\oint m\vec{v}\cdot d\vec{l} = mv\times(2\pi R) = qBR\times 2\pi R = 2\pi qBR^2$$

Vector potential term:

Use symmetric gauge $\vec{A} = \frac{B}{2}(-y, x, 0),$ which gives $\nabla\times\vec{A} = B\hat{z}.$

On the circle $(x, y) = R(\cos\theta, \sin\theta):\ -y\,dx + x\,dy = R^2\,d\theta$

$$\Rightarrow \vec{A}\cdot d\vec{l} = \frac{B}{2}R^2\,d\theta$$

The direction of circulation matters here. Solving $m\dot{\vec{v}} = q\vec{v}\times B\hat{z}$ shows that for $q>0,\ B>0,$ the particle traverses the circle CLOCKWISE (the magnetic force curves the velocity in that sense).

Moving in the direction of motion (clockwise), θ decreases by 2π over one loop:

$$\oint d\theta = -2\pi$$

$$q\oint \vec{A}\cdot d\vec{l} = q\cdot\frac{B}{2}R^2\cdot(-2\pi) = -\pi qBR^2$$

Total:

$$\oint_C \vec{p}\cdot d\vec{l} = 2\pi qBR^2 - \pi qBR^2 = \pi qBR^2$$

Corrected answer: Option 1, $qB\pi R^2$

Q36.

Using cylindrical coordinates, since the wire is forced to rotate at constant angular velocity, $\dot{\phi} = \omega$ is a rheonomic (time-dependent) constraint, and the wire shape gives $z = cr^2$. The only free coordinate is r.

Position and velocity:

$$v^2 = \dot{r}^2 + r^2\omega^2 + \dot{z}^2, \qquad \dot{z} = 2cr\dot{r}$$

$$v^2 = \dot{r}^2(1 + 4c^2r^2) + r^2\omega^2$$

Lagrangian (effectively single-DOF in r):

$$L = \frac{1}{2}M\left[(1 + 4c^2r^2)\dot{r}^2 + r^2\omega^2\right] - Mgcr^2$$

Angular momentum:

$$L_z = Mr^2\omega$$

$$\frac{dL_z}{dt} = 2Mr\dot{r}\,\omega \neq 0 \text{ whenever } \dot{r} \neq 0$$

This is because the wire (via the motor maintaining constant ω) must exert a tangential constraint force on the bead as r changes, providing external torque $\Rightarrow$ angular momentum is NOT conserved.

Energy:

Using the equation of motion derived from L, one finds

$$\frac{dE}{dt} = \frac{d}{dt}\left[\frac{1}{2}M\left((1 + 4c^2r^2)\dot{r}^2 + r^2\omega^2\right) + Mgcr^2\right] = 2Mr\dot{r}\,\omega^2 \neq 0$$

This nonzero rate reflects the work done by the external agent (motor) that keeps ω constant even as the bead's radial position changes $\Rightarrow$ energy is NOT conserved either.

Correct answer: Option 4, Both energy and angular momentum are not conserved

Q37.

$$\text{Substitute } z = e^{i\theta}, \text{ so } dz = ie^{i\theta}d\theta = iz\,d\theta \Rightarrow d\theta = \frac{dz}{iz}$$

$$\cos\theta = \frac{z + z^{-1}}{2}$$

$$5 - 4\cos\theta = 5 - 4\cdot\frac{z+z^{-1}}{2} = 5 - 2z - \frac{2}{z} = \frac{5z - 2z^2 - 2}{z}$$

$$I = \oint \frac{1}{5-4\cos\theta}\,d\theta = \oint \frac{z}{-2z^2+5z-2}\cdot\frac{dz}{iz} = \oint \frac{dz}{i(-2z^2+5z-2)}$$

$$= \frac{i}{2}\oint \frac{dz}{z^2 - \frac{5}{2}z + 1}$$

$$\text{Factor the denominator: } z^2 - \frac{5}{2}z + 1 = (z-2)\left(z - \frac{1}{2}\right)$$

$$\text{Poles at } z = 2 \text{ (outside } |z| = 1\text{) and } z = \frac{1}{2} \text{ (inside } |z| = 1\text{)}$$

$$\text{Only the pole at } z = \frac{1}{2} \text{ contributes. By the residue theorem:}$$

$$\text{Res}_{z=1/2}\frac{1}{(z-2)(z-1/2)} = \frac{1}{\frac{1}{2}-2} = \frac{1}{-\frac{3}{2}} = -\frac{2}{3}$$

$$I = \frac{i}{2}\times 2\pi i \times \left(-\frac{2}{3}\right) = \frac{i}{2}\times\left(-\frac{4\pi i}{3}\right) = \frac{-4\pi i^2}{6} = \frac{4\pi}{6} = \frac{2\pi}{3}$$

Correct answer: Option 1, $\frac{2\pi}{3}$

Q38.

$$\text{Energy eigenvalues of an infinite square well: } E_n = n^2 E_1$$

$$E_1 = 1.2 \text{ eV}, \quad E_2 = 4E_1 = 4.8 \text{ eV}, \quad E_4 = 16E_1 = 19.2 \text{ eV}$$

$$\text{The given state: } \psi = 3\psi_1 + 2\psi_2 + \sqrt{3}\,\psi_4$$

$$\text{Normalization (sum of squares of coefficients):}$$
$$N = 3^2 + 2^2 + (\sqrt{3})^2 = 9 + 4 + 3 = 16$$

$$\text{Expectation value of energy:}$$
$$\langle E\rangle = \frac{3^2E_1 + 2^2E_2 + (\sqrt{3})^2E_4}{N} = \frac{9E_1 + 4E_2 + 3E_4}{16}$$
$$= \frac{9(1.2) + 4(4.8) + 3(19.2)}{16} = \frac{10.8 + 19.2 + 57.6}{16} = \frac{87.6}{16}$$
$$\langle E\rangle = 5.475 \text{ eV} \approx 5.5 \text{ eV}$$

Correct answer: Option 1, 5.5 eV

Q39.

Since $J = K = 1$ for every flip-flop, each one operates in TOGGLE mode: its output complements every time a triggering clock edge arrives.

The clock pulse is applied only to $Q_0's$ flip-flop; $Q_0's$ output feeds the clock of $Q_1's$ flip-flop, and $Q_1's$ output feeds the clock of $Q_2's$ flip-flop. This is the standard asynchronous (ripple) binary counter configuration, where $Q_2Q_1Q_0$ counts up in binary with each external clock pulse, incrementing by 1 modulo 8.

$$\text{Initial state: } Q_2Q_1Q_0 = 100_2 = 4 \text{ (decimal)}$$

$$\text{After 9 clock pulses:}$$
$$\text{Final count} = (4 + 9) \bmod 8 = 13 \bmod 8 = 5$$

$$5 \text{ in binary} = 101$$

$$\text{Tracking explicitly: } 100 \to 101 \to 110 \to 111 \to 000 \to 001 \to 010 \to 011 \to 100 \to 101$$
$$\text{(9 transitions from the initial state 100 lead to 101)}$$

Correct answer: Option 3, $Q_2Q_1Q_0 = 101$

Q40.

Normalize the given eigenvectors:

$$\hat{u}_1 = \frac{1}{\sqrt{3}}\begin{pmatrix}1\\1\\1\end{pmatrix}, \quad \hat{u}_2 = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\-1\end{pmatrix}, \quad \hat{u}_3 = \frac{1}{\sqrt{6}}\begin{pmatrix}1\\-2\\1\end{pmatrix}$$

$$\text{Expand } |\psi(0)\rangle = \begin{pmatrix}1\\0\\0\end{pmatrix} \text{ in this eigenbasis:}$$

$$a_1 = \langle \hat{u}_1|\psi(0)\rangle = \frac{1}{\sqrt{3}}, \quad a_2 = \langle \hat{u}_2|\psi(0)\rangle = \frac{1}{\sqrt{2}}, \quad a_3 = \langle \hat{u}_3|\psi(0)\rangle = \frac{1}{\sqrt{6}}$$

$$\text{Check: } a_1^2 + a_2^2 + a_3^2 = \frac{1}{3} + \frac{1}{2} + \frac{1}{6} = 1 \checkmark$$

$$\text{Time evolution: } |\psi(t)\rangle = a_1 e^{-iE_0t/\hbar}\hat{u}_1 + a_2 e^{-i2E_0t/\hbar}\hat{u}_2 + a_3 e^{-i3E_0t/\hbar}\hat{u}_3$$

$$\text{Return amplitude: } A(t) = \langle \psi(0)|\psi(t)\rangle = a_1^2 e^{-iE_0t/\hbar} + a_2^2 e^{-i2E_0t/\hbar} + a_3^2 e^{-i3E_0t/\hbar}$$

$$A(t) = \frac{1}{3}e^{-iE_0t/\hbar} + \frac{1}{2}e^{-i2E_0t/\hbar} + \frac{1}{6}e^{-i3E_0t/\hbar}$$

$$\text{At } t = \frac{\pi\hbar}{2E_0}: \quad \frac{E_0 t}{\hbar} = \frac{\pi}{2}, \quad \frac{2E_0 t}{\hbar} = \pi, \quad \frac{3E_0 t}{\hbar} = \frac{3\pi}{2}$$

$$e^{-i\pi/2} = -i, \qquad e^{-i\pi} = -1, \qquad e^{-i3\pi/2} = i$$

$$A = \frac{1}{3}(-i) + \frac{1}{2}(-1) + \frac{1}{6}(i) = -\frac{1}{2} + i\left(-\frac{1}{3} + \frac{1}{6}\right) = -\frac{1}{2} - \frac{i}{6}$$

$$\text{Probability: } P = |A|^2 = \left(\frac{1}{2}\right)^2 + \left(\frac{1}{6}\right)^2 = \frac{1}{4} + \frac{1}{36} = \frac{9+1}{36} = \frac{10}{36} = \frac{5}{18}$$

Correct answer: Option 3, $\frac{5}{18}$

Q41.

Vanadium: $[Ar]\,3d^3 4s^2$. The $4s$ subshell is completely filled, so it contributes zero net spin and orbital angular momentum. The ground state term is determined entirely by the three $3d$ electrons.

For d electrons, $l = 2,$ so m_l can take values $-2, -1, 0, 1, 2$ (5 orbitals).

Hund's first rule (maximize S):

Place all 3 electrons with parallel spins in different orbitals:

$$S = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{3}{2} \quad \Rightarrow \quad \text{multiplicity} = 2S + 1 = 4$$

Hund's second rule (maximize L, consistent with above):

Fill the highest available m_l values with parallel spin electrons:

$$m_l = 2,\ 1,\ 0 \quad \Rightarrow \quad M_L^{\max} = 2 + 1 + 0 = 3 \quad \Rightarrow \quad L = 3\,(F \text{ term})$$

This gives the term symbol: ^4F

Hund's third rule (determine J):

Since the $3d^3$ subshell is less than half filled (3 electrons out of 10),

$$J = |L - S| = \left|3 - \frac{3}{2}\right| = \frac{3}{2}$$

Therefore the ground state term symbol is $^4F_{3/2}$

Correct answer: Option 3, $^4F_{3/2}$

Q42.

The spherical unit vectors expressed in Cartesian components are:

$$\hat{\theta} = \cos\theta\cos\phi\,\hat{x} + \cos\theta\sin\phi\,\hat{y} - \sin\theta\,\hat{z}$$
$$\hat{\phi} = -\sin\phi\,\hat{x} + \cos\phi\,\hat{y}$$

Differentiate $\hat{\theta}$ with respect to $\phi,$ treating r, θ as fixed:

$$\frac{\partial\hat{\theta}}{\partial\phi} = -\cos\theta\sin\phi\,\hat{x} + \cos\theta\cos\phi\,\hat{y} + 0$$
$$= \cos\theta\,(-\sin\phi\,\hat{x} + \cos\phi\,\hat{y})$$

The bracketed term is exactly $\hat{\phi},$ so

$$\frac{\partial\hat{\theta}}{\partial\phi} = \cos\theta\,\hat{\phi}$$

Correct answer: Option 1, $\cos\theta\,\hat{\phi}$

Q43.

The container is insulated (no heat exchange) and the gas expands into vacuum (free expansion), so no external work is done: $Q = 0,\ W = 0$

$$\Rightarrow\ \Delta U = 0\ \Rightarrow\ U_1 = U_2$$

$$\text{Initial state: } U_1 = \frac{3}{2}RT_1 - \frac{a}{V_1}$$
$$\text{Final state: } U_2 = \frac{3}{2}RT_2 - \frac{a}{V_2}$$

$$\text{Setting } U_1 = U_2:$$
$$\frac{3}{2}RT_1 - \frac{a}{V_1} = \frac{3}{2}RT_2 - \frac{a}{V_2}$$
$$\frac{3}{2}R(T_1 - T_2) = \frac{a}{V_1} - \frac{a}{V_2}$$
$$T_1 - T_2 = \frac{2a}{3R}\left(\frac{1}{V_1} - \frac{1}{V_2}\right)$$
$$T_2 = T_1 - \frac{2a}{3R}\left(\frac{1}{V_1} - \frac{1}{V_2}\right) = T_1 + \frac{2a}{3R}\left(\frac{1}{V_2} - \frac{1}{V_1}\right)$$

Since $V_2 > V_1,$ the bracket is negative, so $T_2 < T_1,$ consistent with the expected cooling on free expansion of an attractive (non-ideal) gas.

Correct answer: Option 3, $T_1 + \frac{2a}{3R}\left(\frac{1}{V_2} - \frac{1}{V_1}\right)$

Q44.

Copper: $[Ar]4s^1 3d^{10}$. The $3d^{10}$ subshell is completely filled, so it contributes zero net orbital and spin angular momentum.
The single $4s^1$ electron alone determines the atom's magnetic properties.

$$\text{For this single s-electron: } L = 0, \quad S = \frac{1}{2}, \quad J = \frac{1}{2}$$

$$\text{Landé g-factor: } g_J = 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)}$$
$$= 1 + \frac{\frac{3}{4} + \frac{3}{4} - 0}{2 \times \frac{3}{4}} = 1 + \frac{\frac{3}{2}}{\frac{3}{2}} = 1 + 1 = 2$$

The force on the atom in the Stern-Gerlach apparatus is
$$F_z = -\mu_z \frac{\partial B_z}{\partial z} = g_J\, m_J\, \mu_B\, \frac{\partial B_z}{\partial z}$$

Since $J = \frac{1}{2},\ m_J = \pm\frac{1}{2}$. The magnitude of the force is
$$|F_z| = g_J\, |m_J|\, \mu_B\, \frac{\partial B_z}{\partial z} = 2 \times \frac{1}{2} \times \mu_B \times 10^3 = 10^3 \mu_B \text{ (in appropriate units)}$$

Q45.

Ten degenerate levels, two particles distributed among them.

Distinguishable particles (Maxwell-Boltzmann counting):

Each particle independently chooses any of the 10 levels:

$$\Omega_C = 10 \times 10 = 100$$

Identical bosons:

Multiple bosons can occupy the same level. The number of ways to distribute N identical bosons among g levels is $\binom{N+g-1}{N}$

$$\Omega_B = \binom{2+10-1}{2} = \binom{11}{2} = \frac{11 \times 10}{2} = 55$$

Identical fermions:

By the Pauli exclusion principle, no two fermions can occupy the same level, so the number of ways is $\binom{g}{N}$

$$\Omega_F = \binom{10}{2} = \frac{10 \times 9}{2} = 45$$

$$\text{Summary: } \Omega_B = 55, \quad \Omega_F = 45, \quad \Omega_C = 100$$

Correct answer: Option 1, $\Omega_B = 55,\ \Omega_F = 45,\ \Omega_C = 100$

Q46.

$$f(x) = x^4, \quad h = 1, \quad \text{nodes: } x = 0, 1, 2, 3, 4$$

$$f(0) = 0, \quad f(1) = 1, \quad f(2) = 16, \quad f(3) = 81, \quad f(4) = 256$$

Trapezoidal rule:

$$I_t = \frac{h}{2}\Big[f(0) + f(4) + 2\big(f(1) + f(2) + f(3)\big)\Big]$$

$$= \frac{1}{2}\Big[0 + 256 + 2(1 + 16 + 81)\Big] = \frac{1}{2}[256 + 196] = \frac{452}{2} = 226.0$$

Simpson's 1/3rd rule (n=4 intervals, even, applicable):

$$I_s = \frac{h}{3}\Big[f(0) + f(4) + 4\big(f(1) + f(3)\big) + 2f(2)\Big]$$

$$= \frac{1}{3}\Big[0 + 256 + 4(1 + 81) + 2(16)\Big] = \frac{1}{3}[256 + 328 + 32] = \frac{616}{3} = 205.3$$

(For comparison, the exact value is $\int_0^4 x^4\,dx = \frac{4^5}{5} = 204.8$, showing Simpson's rule is considerably more accurate than the trapezoidal rule.)

Correct answer: Option 1, $I_s = 205.3,\ I_t = 226.0$

Q47.

Many low-frequency noise sources, particularly $1/f$ (flicker) noise and slow instrumental drift, have power that is concentrated heavily at low frequencies and decreases as frequency increases.

By modulating the signal, i.e. multiplying it with a carrier wave, the information content is shifted from near DC (low frequency) to a much higher carrier frequency band, where $1/f$ noise power is negligible.

After the shifted signal is detected (demodulated) back down, it is now far removed spectrally from the noisy low-frequency region, so drift and $1/f$ noise contribute much less to the measurement.

In contrast, white noise (thermal/Johnson noise, shot noise) has a flat power spectral density across all frequencies, so shifting the signal to a higher frequency does not help reduce these noise sources at all.

This is precisely the working principle behind lock-in amplifier techniques used in precision measurements.

Correct answer: Option 2, reduce the effects of $1/f$-noise and drift

Q48.

In the center of mass frame (rest frame of K^0), the two pions are emitted back-to-back with equal and opposite momenta of magnitude p, and by energy-momentum conservation each pion carries energy

$$E_\pi = \frac{M_K c^2}{2}$$

$$E_\pi = \frac{495}{2} = 247.5 \text{ MeV}$$

Using the relativistic energy-momentum relation for each pion:

$$E_\pi^2 = (pc)^2 + (m_\pi c^2)^2$$

$$(pc)^2 = E_\pi^2 - (m_\pi c^2)^2 = (247.5)^2 - (135)^2$$

$$= 61256.25 - 18225 = 43031.25$$

$$pc = \sqrt{43031.25} \approx 207.4 \text{ MeV}$$

Correct answer: Option 1, 207 MeV/c

Q49.

For a uniform distribution on $(0, 1)$, the variance of a single random variable X is

$$\sigma_X^2 = \frac{(b-a)^2}{12} = \frac{1}{12}$$

By the Central Limit Theorem, the standard deviation of the sample mean $\bar{X}$ of n independent draws is

$$\sigma_{\bar{X}} = \frac{\sigma_X}{\sqrt{n}}$$

Here each experiment computes the mean of $n = 100$ random numbers, and this is repeated 100 times, so $n = 100$ is the relevant sample size for the standard deviation of the means.

$$\sigma_{\bar{X}} = \frac{\sqrt{1/12}}{\sqrt{100}} = \frac{1}{\sqrt{1200}}$$

$$\sqrt{1200} = \sqrt{400 \times 3} = 20\sqrt{3}$$

$$\sigma_{\bar{X}} = \frac{1}{20\sqrt{3}}$$

Correct answer: Option 1, $\frac{1}{20\sqrt{3}}$

Q50.

The changing magnetic field $\vec{B} = \beta t \hat{z}$ induces an azimuthal electric field via Faraday's law: $\oint \vec{E} \cdot d\vec{l} = -\dfrac{d\Phi_B}{dt} = -\pi R^2 \beta$

$$E_\phi (2\pi R) = -\pi R^2 \beta \;\Rightarrow\; E_\phi = -\frac{R\beta}{2}$$

This gives a tangential force $F_\phi = qE_\phi = -\dfrac{qR\beta}{2}$, producing an angular acceleration about the pivot:

$$mR^2 \ddot{\phi} = RF_\phi = -\frac{qR^2\beta}{2} \;\Rightarrow\; \ddot{\phi} = -\frac{q\beta}{2m} \text{ (constant)}$$

$$\text{Integrating: } \omega(t) = \dot{\phi}(t) = -\frac{q\beta t}{2m}$$

The rotating charge, moving with velocity $\vec{v} = R\omega\hat{\phi}$, also experiences the instantaneous Lorentz force from $\vec{B} = \beta t \hat{z}$:

$$\vec{F}_B = q\vec{v} \times \vec{B} = qR\omega\beta t \,(\hat{\phi} \times \hat{z}) = qR\omega\beta t \,\hat{r}$$

$$\text{Substituting } \omega = -\frac{q\beta t}{2m} : \quad \vec{F}_B = -\frac{q^2 R\beta^2 t^2}{2m}\,\hat{r} \quad \text{(radially INWARD)}$$

Newton's second law in the radial direction (net inward force = centripetal requirement $m\omega^2 R$):

$$T_{\text{inward}} + \frac{q^2 R\beta^2 t^2}{2m} = m\omega^2 R = mR\left(\frac{q\beta t}{2m}\right)^2 = \frac{q^2\beta^2 t^2 R}{4m}$$

$$T_{\text{inward}} = \frac{q^2\beta^2 t^2 R}{4m} - \frac{q^2\beta^2 t^2 R}{2m} = -\frac{q^2\beta^2 t^2 R}{4m}$$

The negative sign means the rod must actually push the particle outward rather than pull it inward — i.e., the rod is under compression.

Correct answer: Option 3, Compressive and of magnitude $q^2\beta^2 t^2 R/(4m)$

Q51.

$$\text{Unperturbed eigenstates of } H_0 : \; \psi_m(\phi) = \frac{1}{\sqrt{2\pi}} e^{im\phi}, \quad E_m = \frac{\hbar^2 m^2}{2mR^2}$$

The first excited state has $m = \pm 1$ (both give the same energy E_1), so this level is two-fold degenerate: basis $\{|1\rangle, |-1\rangle\}$

$$\text{Write the perturbation as } H' = V\cos(2\phi) = \frac{V}{2}\left(e^{i2\phi} + e^{-i2\phi}\right)$$

Degenerate perturbation theory requires diagonalizing H' within the degenerate subspace. Compute the matrix elements

$$\langle m|H'|n\rangle = \frac{1}{2\pi}\int_0^{2\pi} e^{-im\phi} H'(\phi) e^{in\phi}\, d\phi$$

Diagonal elements:

$$\langle 1|H'|1\rangle = \frac{V}{2}\cdot\frac{1}{2\pi}\int (e^{i2\phi} + e^{-i2\phi})\, d\phi = 0, \qquad \langle -1|H'|-1\rangle = 0$$

Off-diagonal elements:

$$\langle 1|H'|-1\rangle = \frac{V}{2}\cdot\frac{1}{2\pi}\int e^{-i\phi}(e^{i2\phi} + e^{-i2\phi})e^{-i\phi}\, d\phi = \frac{V}{2}\cdot\frac{1}{2\pi}\int 1\, d\phi = \frac{V}{2}$$

$$\langle -1|H'|1\rangle = \frac{V}{2} \quad \text{(Hermitian conjugate, real here)}$$

The perturbation matrix in the degenerate subspace is

$$H' \to \begin{pmatrix} 0 & V/2 \\ V/2 & 0 \end{pmatrix}$$

$$\text{Diagonalizing this } 2\times 2 \text{ matrix gives eigenvalues } \pm\frac{V}{2}$$

$$\text{Energy splitting} = \frac{V}{2} - \left(-\frac{V}{2}\right) = V$$

Correct answer: Option 2, V

Q52.

$$G = apq + bp^2t + \frac{ct}{q^2}, \qquad H = \frac{p^2}{2} - \frac{1}{2q^2}$$

For G to be a constant of motion: $\dfrac{dG}{dt} = \dfrac{\partial G}{\partial t} + \{G, H\} = 0$

$$\frac{\partial G}{\partial t} = bp^2 + \frac{c}{q^2}$$

Poisson bracket: $\{G, H\} = \dfrac{\partial G}{\partial q}\dfrac{\partial H}{\partial p} - \dfrac{\partial G}{\partial p}\dfrac{\partial H}{\partial q}$

$$\frac{\partial G}{\partial q} = ap - \frac{2ct}{q^3}, \qquad \frac{\partial G}{\partial p} = aq + 2bpt$$

$$\frac{\partial H}{\partial p} = p, \qquad \frac{\partial H}{\partial q} = \frac{1}{q^3}$$

$$\{G, H\} = \left(ap - \frac{2ct}{q^3}\right)p - (aq + 2bpt)\frac{1}{q^3}$$

$$= ap^2 - \frac{2ctp}{q^3} - \frac{a}{q^2} - \frac{2bpt}{q^3}$$

Adding the two pieces:

$$\frac{dG}{dt} = (a+b)p^2 + \frac{c-a}{q^2} - \frac{2(c+b)tp}{q^3}$$

For this to vanish identically for all $p, q, t,$ each independent coefficient must vanish:

$$a + b = 0, \qquad c - a = 0, \qquad c + b = 0$$

From the first two: $b = -a,\ c = a$. The third equation, $c + b = a - a = 0$, is then automatically satisfied.

$$\Rightarrow\ a = c = -b$$

Correct answer: Option 2, $a = -b = c$

Q53. (No solution uploaded yet.)

Q54.

The op-amp is in negative feedback (output feeds back to the inverting input through the zener diode), so the virtual short condition applies:

$$V_- = V_+ = V_{in} = 2.0 \text{ V}$$

Since the op-amp's inverting input draws no current, the current flowing through the 1k resistor to ground must be supplied entirely through the zener diode from V_{out}:

$$I = \frac{V_-}{1\text{k}\Omega} = \frac{2.0 \text{ V}}{1000 \ \Omega} = 2 \text{ mA}$$

This current flows from V_{out} through the zener diode (operating in reverse breakdown at its rated 5.1 V) down to the V_- node, so

$$V_{out} = V_- + V_Z$$

$$V_{out} = 2.0 \text{ V} + 5.1 \text{ V} = 7.1 \text{ V}$$

Correct answer: Option 4, 7.1 V

Q55.

$$\hat{H} = \alpha \hat{L}^2 + \beta \hbar \hat{L}_z$$

$$\text{Acting on the state } |l, m\rangle: \quad \hat{L}^2 |l, m\rangle = \hbar^2 l(l+1) |l, m\rangle, \quad \hat{L}_z |l, m\rangle = \hbar m |l, m\rangle$$

$$E_{l,m} = \alpha \hbar^2 l(l+1) + \beta \hbar^2 m, \qquad -l \le m \le l$$

$$\text{For a fixed } l, \text{ since } \beta > 0, \text{ the energy is minimized by taking the most negative allowed } m, \text{ i.e. } m = -l:$$

$$E_l^{\min} = \alpha \hbar^2 l(l+1) - \beta \hbar^2 l = \hbar^2 l \big[\alpha(l+1) - \beta\big]$$

Compare the ground candidates:

$$l = 0: \quad E_0 = 0$$

$$l = 1,\ m = -1: \quad E_1 = \hbar^2 (2\alpha - \beta)$$

The $l = 1$ state is lower in energy than $l = 0$ if and only if

$$2\alpha - \beta < 0 \ \Rightarrow\ \alpha < \frac{\beta}{2}$$

Checking higher l does not give a lower minimum in this regime, since the quadratic-in-l growth from $\alpha l(l+1)$ eventually dominates the linear $-\beta l$ term; the state $l = 1, m = -1$ is the global minimum for small $\alpha < \beta/2$.

So under the condition $\alpha < \dfrac{\beta}{2}$, the lowest energy state is $l = 1,\ m = -1$

Correct answer: Option 3, $l = 1, m = -1$ for $\alpha < \dfrac{\beta}{2}$

Q56.

In equilibrium, the rate of absorption (stimulated upward transitions) must equal the rate of downward transitions (spontaneous + stimulated).

Since both levels are non-degenerate, the Einstein B-coefficients for absorption and stimulated emission are equal: $B_{12} = B_{21} = B$

$$\text{Rate of absorption } (1 \to 2): \quad N_1 B \rho(\nu, T)$$

$$\text{Rate of emission } (2 \to 1): \quad N_2 A + N_2 B \rho(\nu, T)$$

Setting these equal (detailed balance at equilibrium):

$$N_1 B \rho = N_2 A + N_2 B \rho$$

$$B\rho(N_1 - N_2) = N_2 A$$

$$\rho(\nu, T) = \frac{A}{B} \cdot \frac{N_2}{N_1 - N_2}$$

Correct answer: Option 1, $\frac{A}{B}\left(\frac{N_2}{N_1 - N_2}\right)$

Q57.

Concept: First-order perturbation theory

$$E_n^{(1)} = \langle \psi_n | H' | \psi_n \rangle = \int_0^{2L} \psi_n^*(x)\, H'(x)\, \psi_n(x)\, dx$$

For $H' = V_0 L\, \delta(x - x_0),$ the sifting property gives

$$E_n^{(1)} = V_0 L\, |\psi_n(x_0)|^2$$

Wavefunction of the well (width $a = 2L$):

$$\psi_n(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right) = \sqrt{\frac{1}{L}} \sin\left(\frac{n\pi x}{2L}\right)$$

First excited state: $n = 2$

$$\psi_2(x) = \sqrt{\frac{1}{L}} \sin\left(\frac{2\pi x}{2L}\right) = \sqrt{\frac{1}{L}} \sin\left(\frac{\pi x}{L}\right)$$

Evaluate at $x_0 = \dfrac{3L}{2}$:

$$\psi_2\left(\frac{3L}{2}\right) = \sqrt{\frac{1}{L}} \sin\left(\frac{3\pi}{2}\right) = -\sqrt{\frac{1}{L}}$$

$$\left|\psi_2\left(\frac{3L}{2}\right)\right|^2 = \frac{1}{L}$$

First-order energy correction:

$$E_2^{(1)} = V_0 L \cdot \frac{1}{L} = V_0$$

(Note: $x_0 = 3L/2$ is not a node of $\psi_2,$ since nodes occur at $x = 0, L, 2L,$ so a nonzero correction is expected, ruling out option 4.)

Correct answer: Option 2, $E_2^{(1)} = V_0$

Q58.

Concept: Cutoff frequency of rectangular waveguide

$$f_{mn} = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

where $a = 3 \text{ cm} = 0.03 \text{ m},\ b = 2 \text{ cm} = 0.02 \text{ m},$
and for $TE_{mn}, m, n = 0, 1, 2, \ldots$ (not both zero)

Compute f_{mn} for successive modes ($c/2 = 1.5 \times 10^8$ m/s):

$$TE_{10}:\ f = \frac{c}{2a} = \frac{3 \times 10^8}{2(0.03)} = 5.0 \text{ GHz}$$
$$TE_{01}:\ f = \frac{c}{2b} = \frac{3 \times 10^8}{2(0.02)} = 7.5 \text{ GHz}$$
$$TE_{20}:\ f = \frac{c}{a} = 10.0 \text{ GHz}$$
$$TE_{11}:\ f = 1.5 \times 10^8\sqrt{\left(\frac{1}{0.03}\right)^2 + \left(\frac{1}{0.02}\right)^2} = 9.01 \text{ GHz}$$
$$TE_{21}:\ f = 1.5 \times 10^8\sqrt{\left(\frac{2}{0.03}\right)^2 + \left(\frac{1}{0.02}\right)^2} = 12.5 \text{ GHz}$$
$$TE_{02}:\ f = \frac{c}{b} = 15.0 \text{ GHz}$$
$$TE_{30}:\ f = \frac{3c}{2a} = 15.0 \text{ GHz}$$

Degeneracy check:

All modes below 15 GHz ($5.0,\ 7.5,\ 9.0,\ 10.0,\ 12.5$ GHz) are non-degenerate (each frequency corresponds to a unique (m, n) pair).

At $f = 15.0$ GHz, two distinct modes coincide:

$$TE_{02} \equiv TE_{30}, \quad \text{since } \frac{c}{b} = \frac{3c}{2a} \Rightarrow \frac{1}{0.02} = \frac{3}{2(0.03)} = 50$$

This is the lowest frequency at which mode degeneracy occurs.

Correct answer: Option 4, $f = 15.0$ GHz

Q59.

Concept: Coulomb (electrostatic) energy of a uniformly charged sphere

$$E_C = \frac{3}{5}\frac{1}{4\pi\epsilon_0}\frac{Q^2}{R} = \frac{3}{5}\frac{e^2}{4\pi\epsilon_0}\frac{Z^2}{R}$$

(the Z^2 here effectively becomes $Z(Z-1)$ since a proton does not interact with itself)

Mirror nuclei: ${}^{A}_{Z}X$ and ${}^{A}_{Z-1}Y$

Both have the same mass number $A,$ so the same radius $R = R_0 A^{1/3}$

For mirror nuclei, the proton and neutron numbers swap, giving

$$Z + (Z-1) = A \quad \Rightarrow \quad A = 2Z - 1$$

Difference in Coulomb energy:

$$\Delta E_C = \frac{3}{5}\frac{e^2}{4\pi\epsilon_0 R}\Big[Z(Z-1) - (Z-1)(Z-2)\Big]$$

$$= \frac{3}{5}\frac{e^2}{4\pi\epsilon_0 R}(Z-1)\big[Z-(Z-2)\big] = \frac{3}{5}\frac{e^2}{4\pi\epsilon_0 R}2(Z-1)$$

Alternatively, using the simpler Z^2 form (standard mirror-nuclei approximation):

$$\Delta E_C = \frac{3}{5}\frac{e^2}{4\pi\epsilon_0 R}\Big[Z^2 - (Z-1)^2\Big] = \frac{3}{5}\frac{e^2}{4\pi\epsilon_0 R}(2Z-1)$$

Substitute $R = R_0(2Z-1)^{1/3}$:

$$\Delta E_C = \frac{3}{5}\frac{e^2}{4\pi\epsilon_0 R_0}\frac{(2Z-1)}{(2Z-1)^{1/3}} = \frac{3}{5}\frac{e^2}{4\pi\epsilon_0 R_0}(2Z-1)^{2/3}$$

Insert numbers $\left(\frac{e^2}{4\pi\epsilon_0} = 1.44\ \text{MeV·fm},\ R_0 = 1.5\ \text{fm}\right)$:

$$\Delta E_C = \frac{3}{5}\times\frac{1.44}{1.5}(2Z-1)^{2/3} = 0.576\,(2Z-1)^{2/3}$$

Set equal to given value 3.53 MeV:

$$0.576\,(2Z-1)^{2/3} = 3.53$$

$$(2Z-1)^{2/3} = 6.129$$

$$2Z - 1 = (6.129)^{3/2} = 15.17$$

$$Z = \frac{15.17+1}{2} \approx 8.09$$

$$Z \approx 8$$

Correct answer: Option 2, $Z = 8$

Q60.

Concept: London equations for superconductivity

First London equation (equation of motion for superconducting electrons):

$$m_e \frac{d\vec{v}}{dt} = -e\vec{E} \quad \Rightarrow \quad \frac{\partial \vec{J}}{\partial t} = \frac{ne^2}{m_e}\vec{E}$$

Take curl of both sides:

$$\frac{\partial}{\partial t}(\vec{\nabla} \times \vec{J}) = \frac{ne^2}{m_e}(\vec{\nabla} \times \vec{E})$$

Use Faraday's law:

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\Rightarrow \quad \frac{\partial}{\partial t}(\vec{\nabla} \times \vec{J}) = -\frac{ne^2}{m_e}\frac{\partial \vec{B}}{\partial t}$$

Integrate over time:

$$\vec{\nabla} \times \vec{J} = -\frac{ne^2}{m_e}\vec{B} + (\text{const})$$

The integration constant is set to zero (Meissner effect requires $\vec{B} = 0$ whenever $\vec{J} = 0$ inside the bulk superconductor), giving the **second London equation:**

$$\vec{\nabla} \times \vec{J} = -\frac{ne^2}{m_e}\vec{B}$$

$$\Rightarrow \quad \vec{\nabla} \times \vec{J} + \frac{ne^2}{m_e}\vec{B} = 0$$

(This equation, combined with Ampère's law $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$, leads to the exponential decay of $\vec{B}$ inside a superconductor — the Meissner effect.)

Correct answer: Option 2, $\left(\vec{\nabla} \times \vec{J} + \frac{ne^2}{m_e}\vec{B}\right) = 0$

Q61.

Concept: Cauchy–Riemann equations

For $f = u + iv$ analytic:

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}, \qquad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

Given: $u(x, y) = x^3 - 3xy^2$

$$\frac{\partial u}{\partial x} = 3x^2 - 3y^2$$
$$\frac{\partial u}{\partial y} = -6xy$$

Using CR equation 1:

$$\frac{\partial v}{\partial y} = 3x^2 - 3y^2$$

Integrating with respect to y :

$$v = 3x^2y - y^3 + g(x)$$

Using CR equation 2:

$$\frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y} = 6xy$$

But from above, $\frac{\partial v}{\partial x} = 6xy + g'(x)$

$$\Rightarrow \quad g'(x) = 0 \quad \Rightarrow \quad g(x) = \text{constant}$$

Taking the constant as zero:

$$v(x, y) = 3x^2y - y^3$$

(Check: $u + iv = (x^3 - 3xy^2) + i(3x^2y - y^3) = (x + iy)^3 = z^3$, confirming analyticity.)

Correct answer: Option 3, $v(x, y) = 3x^2y - y^3$

Q62.

Concept: Relativistic transformation of electric and magnetic fields

For a boost with velocity $\vec{v} = v\hat{x},$ the field components split into parts parallel and perpendicular to the boost direction.

Transformation laws:

$$E'_{\parallel} = E_{\parallel}, \qquad B'_{\parallel} = B_{\parallel}$$

$$\vec{E}'_{\perp} = \gamma\left(\vec{E}_{\perp} + \vec{v} \times \vec{B}\right)$$

$$\vec{B}'_{\perp} = \gamma\left(\vec{B}_{\perp} - \frac{\vec{v}}{c^2} \times \vec{E}\right)$$

Given situation:

In frame $S,$ the field of the parallel-plate capacitor is purely electric:

$$\vec{E} = E_y\,\hat{y}, \qquad \vec{B} = 0$$

Boost direction: $\hat{x} \quad \Rightarrow \quad \vec{E}$ is entirely perpendicular to the boost

Apply the perpendicular transformation (with $\vec{B} = 0$):

$$\vec{E}'_{\perp} = \gamma\left(\vec{E}_{\perp} + \vec{v} \times \vec{B}\right) = \gamma\vec{E}_{\perp} = \gamma E_y\,\hat{y}$$

$$\text{where } \gamma = \frac{1}{\sqrt{1 - v^2/c^2}} > 1$$

Interpretation:

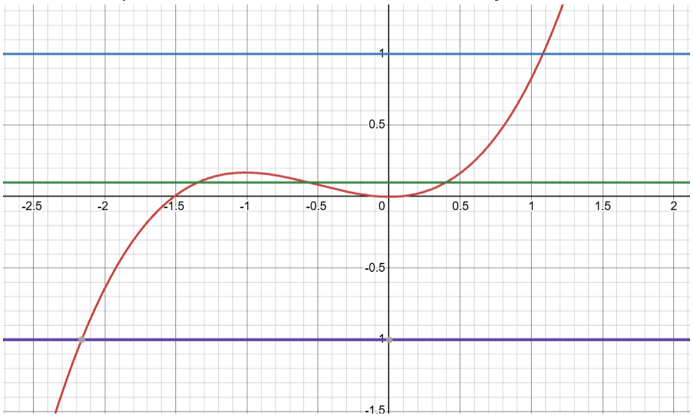
The field remains purely along $\hat{y}$ (direction unchanged), since there is no component along $\hat{x}$ to begin with and no magnetic field to mix in.

The magnitude, however, is scaled up by the factor $\gamma > 1,$ so it increases.

This also matches physical intuition: the moving observer sees the charge density on the plates length-contracted along $x,$ increasing surface charge density $\sigma' = \gamma\sigma,$ and hence a larger field $E' = \sigma'/\epsilon_0.$

Correct answer: Option 1, Direction remains same but magnitude increases

Q63. The plot of the function is given in the image. Blue line is E1 and it is only bound from right. Green line is E2 and it is bound from both sides. Purple line is E3 which is bound only from left.



Q64.

Concept: Flux of a dipole through a spherical cap

For a dipole $\vec{m}$ at the origin, the flux through a spherical cap of radius R (subtending half-angle θ_0 from the dipole axis) is

$$\Phi = \frac{\mu_0 \, m \sin^2\theta_0}{2R}$$

This is obtained from $B_r = \frac{\mu_0}{4\pi}\frac{2m\cos\theta}{R^3}$ integrated over the cap:

$$\Phi = \int_0^{\theta_0} B_r \,(2\pi R^2 \sin\theta)\, d\theta = \frac{\mu_0 m}{R}\int_0^{\theta_0} \sin\theta\cos\theta\, d\theta = \frac{\mu_0 m \sin^2\theta_0}{2R}$$

Geometry of the given loop:

Loop radius $= 3a,$ height $= 4a \;\Rightarrow\;$ distance from origin to loop $= \sqrt{(3a)^2 + (4a)^2} = 5a$

This confirms the hint: use a spherical shell of radius $R = 5a$

$$\sin\theta_0 = \frac{3a}{5a} = \frac{3}{5}, \qquad \cos\theta_0 = \frac{4a}{5a} = \frac{4}{5}$$

Effect of the spinning dipole:

Dipole starts along $+z$ and spins about the x-axis, so

$$\vec{m}(t) = m\cos(\omega t)\,\hat{z} + m\sin(\omega t)\,\hat{y}$$

The loop is centered on and symmetric about the z-axis. By azimuthal symmetry, a dipole component lying in the xy-plane (here, the $\hat{y}$ component) contributes **zero net flux** through a loop that is axially symmetric about z (equal and opposite contributions cancel around the loop).

Only the z-component of $\vec{m}(t),$ namely $m\cos\omega t,$ produces flux through the loop.

Flux through the loop at time t :

$$\Phi(t) = \frac{\mu_0\,[m\cos\omega t]\,\sin^2\theta_0}{2R} = \frac{\mu_0 m\cos\omega t}{2(5a)}\left(\frac{3}{5}\right)^2$$
$$= \frac{\mu_0 m\cos\omega t}{10a}\cdot\frac{9}{25} = \frac{9\mu_0 m}{250a}\cos\omega t$$

Induced EMF (Faraday's law):

$$E = -\frac{d\Phi}{dt} = \frac{9\mu_0 m\omega}{250a}\sin\omega t$$

Correct answer: Option 3, $E = \left(\frac{9\mu_0 m\omega}{250a}\right)\sin\omega t$

Q65. **Concept: Counting independent components via symmetric/antisymmetric index pairs**

$$T_{abcd}, \quad a,b,c,d = 1,\dots,4$$

$$\text{Symmetry: } T_{abcd} = T_{bacd} \text{ (symmetric in } a,b)$$

$$\text{Symmetry: } T_{abcd} = T_{abdc} \text{ (symmetric in } c,d)$$

$$\text{Antisymmetry: } T_{abcd} = -T_{cdab} \text{ (antisymmetric under pair exchange } ab \leftrightarrow cd)$$

Step 1: Reduce to a pair-index object

Since T_{abcd} is symmetric separately in (a,b) and in (c,d), define

$$S_{IJ} \equiv T_{abcd}, \qquad I = (ab),\ J = (cd)$$

where I and J each run over the independent symmetric index-pairs from 4 values.

Step 2: Count symmetric pairs

Number of independent symmetric pairs from $n = 4$ indices:

$$\binom{n+1}{2} = \binom{5}{2} = \frac{4 \cdot 5}{2} = 10$$

So I and J each take 10 possible values $\Rightarrow$ S_{IJ} is a 10×10 matrix

Step 3: Apply the pair-exchange antisymmetry

$$T_{abcd} = -T_{cdab} \quad \Longrightarrow \quad S_{IJ} = -S_{JI}$$

So S_{IJ} is an antisymmetric matrix in the 10-dimensional pair-space

Step 4: Count independent components of an antisymmetric $N \times N$ matrix

$$N = 10: \qquad \frac{N(N-1)}{2} = \frac{10 \times 9}{2} = 45$$

(Note: diagonal terms S_{II} automatically vanish since $S_{II} = -S_{II} \Rightarrow S_{II} = 0$,
consistent with the antisymmetric-matrix count already excluding them.)

Correct answer: Option 2, 45

Q66.

Concept: Newton–Raphson iteration

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$f(x) = (x^2 - 1)(x - 2) = x^3 - 2x^2 - x + 2$$

$$f'(x) = 3x^2 - 4x - 1$$

$$\text{Exact roots: } x = 1,\ -1,\ 2$$

Case 1: $x_0 = 0$

$$f(0) = (0 - 1)(0 - 2) = (-1)(-2) = 2$$

$$f'(0) = 3(0)^2 - 4(0) - 1 = -1$$

$$x_1 = 0 - \frac{2}{-1} = 0 + 2 = 2$$

$$\text{Check: } f(2) = (4 - 1)(2 - 2) = 3 \times 0 = 0 \ \Rightarrow \ x_1 = 2 \text{ is already an exact root}$$

$$\Rightarrow \quad p = 2$$

Case 2: $x_0 = 0.5$

$$f(0.5) = (0.25 - 1)(0.5 - 2) = (-0.75)(-1.5) = 1.125$$

$$f'(0.5) = 3(0.25) - 4(0.5) - 1 = 0.75 - 2 - 1 = -2.25$$

$$x_1 = 0.5 - \frac{1.125}{-2.25} = 0.5 + 0.5 = 1.0$$

$$\text{Check: } f(1) = (1 - 1)(1 - 2) = 0 \times (-1) = 0 \ \Rightarrow \ x_1 = 1 \text{ is already an exact root}$$

$$\Rightarrow \quad q = 1$$

Both trial points happen to land exactly on a root in a single Newton step, since the tangent line construction coincides with the exact root location here.

Correct answer: Option 3, $p = 2,\ q = 1$

Q67.

Concept: WKB tunneling probability through a triangular barrier

$$p(\mathcal{E}) = \exp\left(-\frac{2}{\hbar}\int_0^{x_0}\sqrt{2m\left[V(x)-E_F\right]}\,dx\right)$$

Barrier profile (energy measured from $E_F = 0$):

$$V(x) - E_F = e(\phi_0 - x\mathcal{E})$$

(the work function ϕ_0 is a potential, so the actual barrier energy is $e\phi_0$, consistent with the given $\lambda = \sqrt{2me\phi_0^3/\hbar^2}$)

Classical turning point (where barrier meets E_F): $x_0 = \dfrac{\phi_0}{\mathcal{E}}$

Evaluate the WKB integral:

$$I = \int_0^{x_0}\sqrt{2me(\phi_0 - x\mathcal{E})}\,dx$$

Substitute $u = \phi_0 - x\mathcal{E},\ du = -\mathcal{E}\,dx$:

$$I = \frac{1}{\mathcal{E}}\int_0^{\phi_0}\sqrt{2me\,u}\,du = \frac{\sqrt{2me}}{\mathcal{E}}\cdot\frac{2}{3}\phi_0^{3/2} = \frac{2}{3\mathcal{E}}\sqrt{2me\phi_0^3}$$

Tunneling probability:

$$p(\mathcal{E}) = \exp\left(-\frac{2}{\hbar}I\right) = \exp\left(-\frac{4}{3\mathcal{E}\hbar}\sqrt{2me\phi_0^3}\right)$$

Recognizing $\lambda = \dfrac{\sqrt{2me\phi_0^3}}{\hbar}$:

$$p(\mathcal{E}) = \exp\left(-\frac{4\lambda}{3\mathcal{E}}\right)$$

Compute the required ratio:

$$\frac{p(\mathcal{E}=2\lambda)}{p(\mathcal{E}=\lambda)} = \frac{\exp\left(-\dfrac{4\lambda}{3(2\lambda)}\right)}{\exp\left(-\dfrac{4\lambda}{3\lambda}\right)} = \frac{\exp(-2/3)}{\exp(-4/3)} = \exp\left(\frac{4}{3}-\frac{2}{3}\right) = \exp\left(\frac{2}{3}\right)$$

$$\exp(2/3) \approx 1.947 \approx 1.94$$

Correct answer: Option 3, 1.94

Q68.

Concept: Linear stability analysis of a nonlinear autonomous system

$$\ddot{x} - \dot{x} + x^2 - 2x = 0 \quad \Rightarrow \quad \ddot{x} = \dot{x} - x^2 + 2x$$

Convert to a first-order system:

$$x_1 = x, \qquad x_2 = \dot{x}$$

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = x_2 - x_1^2 + 2x_1$$

Fixed points: set $\dot{x}_1 = \dot{x}_2 = 0$

$$x_2 = 0, \qquad -x_1^2 + 2x_1 = 0 \;\Rightarrow\; x_1(2 - x_1) = 0 \;\Rightarrow\; x_1 = 0,\, 2$$

We focus on the origin $(0, 0)$

Linearize: compute the Jacobian

$$J = \begin{pmatrix} \dfrac{\partial \dot{x}_1}{\partial x_1} & \dfrac{\partial \dot{x}_1}{\partial x_2} \\ \dfrac{\partial \dot{x}_2}{\partial x_1} & \dfrac{\partial \dot{x}_2}{\partial x_2} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 2 - 2x_1 & 1 \end{pmatrix}$$

At the origin $(x_1 = 0)$:

$$J\big|_{(0,0)} = \begin{pmatrix} 0 & 1 \\ 2 & 1 \end{pmatrix}$$

Trace and determinant:

$$\mathrm{Tr}(J) = 0 + 1 = 1, \qquad \det(J) = (0)(1) - (1)(2) = -2$$

Eigenvalues:

$$\lambda^2 - (\mathrm{Tr}\, J)\lambda + \det J = 0 \quad \Rightarrow \quad \lambda^2 - \lambda - 2 = 0$$

$$(\lambda - 2)(\lambda + 1) = 0 \quad \Rightarrow \quad \lambda_1 = 2, \quad \lambda_2 = -1$$

Classification:

Since $\det J < 0$, the eigenvalues are real with opposite signs
$\Rightarrow$ the origin is a **saddle point (hyperbolic fixed point)**

Because one eigenvalue is positive ($\lambda_1 = 2 > 0$), trajectories grow along that direction, so the fixed point is unstable overall.

Correct answer: Option 3, unstable hyperbola (saddle point)

Q69.

Concept: Angular momentum, parity, and Bose symmetry in two-pion decays

$$\rho^0: \quad J^{PC} = 1^{--}, \qquad \pi: \quad J^P = 0^-$$

Angular momentum conservation:

Each pion has spin 0, so the only angular momentum in the final state is the orbital angular momentum L between the two pions.

Conservation of total angular momentum requires

$$L = J_{\rho^0} = 1$$

Parity conservation:

Intrinsic parity of the two-pion system: $P_\pi \times P_\pi = (-1) \times (-1) = +1$

Orbital contribution: $(-1)^L$

Total parity of final state: $P_f = (+1)(-1)^L = (-1)^L = (-1)^1 = -1$

This matches $P_{\rho^0} = -1$. **Parity is satisfied for** $L = 1$ in both channels.

Process (A): $\rho^0 \to \pi^+\pi^-$

π^+ and π^- are distinguishable (particle–antiparticle), so there is no exchange-symmetry restriction on L.

$L = 1$ satisfies both angular momentum and parity conservation.

$\Rightarrow$ **Process (A) is allowed**

Process (B): $\rho^0 \to \pi^0\pi^0$

Here the two final-state particles are **identical bosons**.

Exchanging the two π^0's is equivalent to $\vec{r} \to -\vec{r}$, which introduces a factor $(-1)^L$

Since the pions are identical bosons, the total wavefunction (purely spatial, as pion spin is zero) must be **symmetric** under this exchange:

$$(-1)^L = +1 \quad \Rightarrow \quad L \text{ must be even}$$

But angular momentum conservation requires $L = 1$ (odd)

$\Rightarrow$ **Contradiction: Process (B) is forbidden by Bose symmetry**

(This is the same reason $\rho^0 \to \pi^0\pi^0$ is never observed experimentally, while $\rho^0 \to \pi^+\pi^-$ is the dominant decay mode.)

Correct answer: Option 3, Only process (A) is allowed

Q70.

Concept: Binomial probability via Stirling's approximation, worked in $\ln P$

$$p = 0.6,\ q = 0.4,\ N = 1000 \text{ steps}$$

$$n_R = 600,\ n_L = 400 \quad (\text{since } m = 200 \Rightarrow n_R = \tfrac{N+m}{2},\ n_L = \tfrac{N-m}{2})$$

$$P = \binom{N}{n_R} p^{n_R} q^{n_L} = \frac{N!}{n_R!\,n_L!}\, p^{n_R} q^{n_L}$$

Take the natural log:

$$\ln P = \ln N! - \ln n_R! - \ln n_L! + n_R \ln p + n_L \ln q$$

Apply Stirling's formula $\ln n! \approx n \ln n - n + \frac{1}{2}\ln(2\pi n)$:

$$\ln N! \approx N \ln N - N + \tfrac{1}{2}\ln(2\pi N)$$
$$\ln n_R! \approx n_R \ln n_R - n_R + \tfrac{1}{2}\ln(2\pi n_R)$$
$$\ln n_L! \approx n_L \ln n_L - n_L + \tfrac{1}{2}\ln(2\pi n_L)$$

Substitute and simplify (the $-N, +n_R, +n_L$ terms cancel since $N = n_R + n_L$):

$$\ln P = N \ln N - n_R \ln n_R - n_L \ln n_L + \frac{1}{2}\ln\left(\frac{N}{2\pi\, n_R n_L}\right) + n_R \ln p + n_L \ln q$$

Group logarithms to isolate the exponential (large-deviation) part:

$$N \ln N - n_R \ln n_R - n_L \ln n_L + n_R \ln p + n_L \ln q$$
$$= -n_R \ln\left(\frac{n_R}{Np}\right) - n_L \ln\left(\frac{n_L}{Nq}\right)$$

Insert numbers:

$$Np = 1000(0.6) = 600 = n_R, \qquad Nq = 1000(0.4) = 400 = n_L$$

$$\Rightarrow \quad \frac{n_R}{Np} = 1, \quad \frac{n_L}{Nq} = 1 \quad \Rightarrow \quad \ln(1) = 0 \text{ for both terms}$$

So the entire exponential part vanishes (walker sits exactly at the mean), leaving only the prefactor:

$$\ln P \approx \frac{1}{2}\ln\left(\frac{N}{2\pi\, n_R n_L}\right) = \frac{1}{2}\ln\left(\frac{1000}{2\pi(600)(400)}\right) = \frac{1}{2}\ln\left(\frac{1}{1508}\right)$$
$$= -\frac{1}{2}\ln(1508) = -\frac{1}{2}(7.319) = -3.659$$

Exponentiate:

$$P = e^{-3.659} \approx 0.0257$$

Correct answer: Option 1, 0.026

Q71.

Concept: Structure factor of the NaCl (rock-salt) structure

KCl has FCC lattice with a two-ion basis: K^+ at $(0,0,0)$, $\quad Cl^-$ at $\left(\frac{1}{2},0,0\right)$

Structure factor:

$$F(hkl) = f_{K^+}\sum_{\text{fcc}} e^{i\vec{G}\cdot\vec{r}_j} + f_{Cl^-}\sum_{\text{fcc}} e^{i\vec{G}\cdot(\vec{r}_j+\hat{x}/2)}$$

$$= \left[4\,(\text{FCC geometric factor, nonzero only if } h,k,l \text{ all even or all odd})\right] \times \left[f_{K^+} + f_{Cl^-}\,e^{i\pi(h+k+l)}\right]$$

Given: $f_{K^+} = f_{Cl^-} = f \quad$ (since K^+ and Cl^- are isoelectronic, both 18-electron ions)

$$F(hkl) = 4f\left[1 + e^{i\pi(h+k+l)}\right] = \begin{cases} 8f, & h+k+l \text{ even} \\ 0, & h+k+l \text{ odd} \end{cases}$$

Combine with the FCC condition (h,k,l all even or all odd):

- All h,k,l odd $\Rightarrow$ $h+k+l = \text{odd}$ $\Rightarrow$ $F = 0$ $\quad$ (**forbidden**)
- All h,k,l even $\Rightarrow$ $h+k+l = \text{even}$ $\Rightarrow$ $F = 8f$ $\quad$ (**allowed**)

So because the two ions have identical form factors, the "odd" FCC reflections exactly cancel, and KCl diffracts as if it were a simple cubic lattice: only **all-even** (h,k,l) reflections survive.

Order allowed peaks by $\sin^2\theta \propto (h^2+k^2+l^2)$:

$$(200):\ 4,\quad (220):\ 8,\quad (222):\ 12,\quad (400):\ 16$$

(note: (311), though satisfying the FCC all-odd rule, has $h+k+l = 5$ odd $\Rightarrow F = 0$, excluded)

Correct answer: Option 4, $(200), (220), (222), (400)$

Q72.

Concept: Schmidt (single-particle) magnetic moment

For a nucleus with one unpaired nucleon in state $(l, j),$ the Schmidt formula gives

$$\text{For } j = l + \tfrac{1}{2}: \qquad \mu = g_l\left(j - \tfrac{1}{2}\right) + \tfrac{1}{2} g_s$$

$$\text{For } j = l - \tfrac{1}{2}: \qquad \mu = \frac{j}{j+1}\left[g_l\left(j + \tfrac{3}{2}\right) - \tfrac{1}{2} g_s\right]$$

Identify the valence nucleon in $^{17}_{8}O$:

$$Z = 8 \text{ (protons)}, \qquad N = 9 \text{ (neutrons)}$$

$Z = 8$ is a magic number $\Rightarrow$ proton shell (up to $1p_{1/2}$) is completely filled and paired $\Rightarrow$ protons contribute zero net magnetic moment

$N = 9 = 8 + 1 \Rightarrow$ 8 neutrons fill the closed shell (also paired), and the 9th (odd) neutron occupies the next available level: $1d_{5/2}$

Quantum numbers of the valence neutron:

$$l = 2 \ (d\text{-state}), \qquad j = l + \tfrac{1}{2} = \tfrac{5}{2}$$

Apply Schmidt formula for $j = l + \frac{1}{2}$, using neutron g-factors ($g_l = 0,\ g_s = -3.82$) :

$$\mu = g_l\left(j - \tfrac{1}{2}\right) + \tfrac{1}{2} g_s = 0 \times \left(\frac{5}{2} - \frac{1}{2}\right) + \frac{1}{2}(-3.82)$$

$$= 0 + (-1.91)$$

$$\mu = -1.91\ \mu_N$$

Correct answer: Option 3, $\mu = -1.91\ \mu_N$

Q73.

Concept: Drude model combined with free-electron (Fermi) theory

$$\text{Resistivity: } \rho = \frac{m}{ne^2\tau} \quad \Rightarrow \quad \tau = \frac{m}{ne^2\rho}$$

$$\text{Mean free path: } \ell = v_F\,\tau$$

Fermi velocity from electron density:

$$k_F = (3\pi^2 n)^{1/3}, \qquad v_F = \frac{\hbar k_F}{m}$$

Combine to eliminate m :

$$\ell = v_F\tau = \frac{\hbar k_F}{m}\cdot\frac{m}{ne^2\rho} = \frac{\hbar k_F}{ne^2\rho}$$

Given data (converted to SI):

$$n = 4\times10^{22}\ \text{cm}^{-3} = 4\times10^{28}\ \text{m}^{-3} \quad (\text{monovalent} \Rightarrow \text{electron density} = \text{atomic density})$$

$$\rho = 1\ \mu\Omega\text{-cm} = 1\times10^{-8}\ \Omega\text{-m}$$

$$\hbar = 10^{-34}\ \text{J-s}, \qquad e = 1.6\times10^{-19}\ \text{C}$$

Compute k_F :

$$k_F = (3\pi^2 n)^{1/3} = \left(3\pi^2\times4\times10^{28}\right)^{1/3} = (2.96\times10^{29})^{1/3} \approx 1.06\times10^{10}\ \text{m}^{-1}$$

Compute mean free path:

$$\ell = \frac{\hbar k_F}{ne^2\rho} = \frac{(10^{-34})(1.06\times10^{10})}{(4\times10^{28})(1.6\times10^{-19})^2(10^{-8})}$$

$$= \frac{1.06\times10^{-24}}{1.024\times10^{-17}} \approx 1.03\times10^{-7}\ \text{m}$$

$$\ell \approx 103\ \text{nm} \approx 100\ \text{nm}$$

Correct answer: Option 2, 100 nm

Q74.

Concept: First-order Born approximation for a spherical potential

$$f(\vec{q}) = -\frac{2m}{\hbar^2 q}\int_0^\infty V(r)\, r\sin(qr)\, dr$$

$$q = |\vec{k}' - \vec{k}|, \qquad q^2 = 4k^2\sin^2(\theta/2)$$

Apply to the spherical square well $V(r) = V_0$ for $r \le R$:

$$f(q) = -\frac{2mV_0}{\hbar^2 q}\int_0^R r\sin(qr)\, dr$$

Evaluate the integral exactly:

$$\int_0^R r\sin(qr)\, dr = \frac{\sin(qR)}{q^2} - \frac{R\cos(qR)}{q}$$

Expand for $qR \ll 1$ (leading nontrivial order):

$$\sin(qR) \approx qR - \frac{(qR)^3}{6}, \qquad \cos(qR) \approx 1 - \frac{(qR)^2}{2}$$

$$\frac{\sin(qR)}{q^2} \approx \frac{R}{q} - \frac{R^3 q}{6}, \qquad \frac{R\cos(qR)}{q} \approx \frac{R}{q} - \frac{R^3 q}{2}$$

The R/q terms cancel between the two pieces, leaving

$$\int_0^R r\sin(qr)\, dr \approx \frac{R^3 q}{2} - \frac{R^3 q}{6} = \frac{R^3 q}{3}$$

Substitute back into $f(q)$:

$$f(q) \approx -\frac{2mV_0}{\hbar^2 q}\cdot\frac{R^3 q}{3} = -\frac{2mV_0 R^3}{3\hbar^2}$$

(the leading term is independent of q, i.e. nearly isotropic scattering for small qR)

Differential cross-section:

$$\frac{d\sigma}{d\Omega} = |f(q)|^2 \propto R^6$$

(since $f(q) \propto R^3$ at leading order, and $R_0, m, V_0, \hbar$ are held fixed)

Correct answer: Option 3, R^6

Q75.

Concept: Reduced chi-square for a least-squares fit

$$\chi^2_{\text{reduced}} = \frac{\chi^2}{\nu}, \qquad \nu = N - p$$

where N is the number of data points and p is the number of fitted parameters

Given data:

$$N = 5 \text{ data points}$$

$$\text{Fit function: } y = ax + b \;\Rightarrow\; p = 2 \text{ parameters } (a, b)$$

$$\chi^2 = 5.1$$

Degrees of freedom:

$$\nu = N - p = 5 - 2 = 3$$

Reduced chi-square:

$$\chi^2_{\text{reduced}} = \frac{\chi^2}{\nu} = \frac{5.1}{3} = 1.7$$

Correct answer: Option 4, 1.7